HOMEWORK 7

1. Write out the disjoint cycle decomposition of (1234)(456)(145).

2. a) What is the order of (1234)(567) in S_7?
b) What is the order of a cycle $(a_1, a_2, \dots, a_k)$ of length k?
c) What is the order of a product of disjoint cycles of lengths k_1, k_2, ..., k_r?

3. Find the largest order of an element in S_5.

4. Find all elements in S_5 which commute with the cycle (123).

5. a) We write $a \sim b$ for $a, b \in G$ if a and b are conjugate in G. Prove that $\sim$ is an equivalence relation.
b) Find all conjugacy classes in S_3, and S_4.
c) How many conjugacy classes are there in S_5?

6. (a) Prove that S_n is generated by $(1, 2)$, $(1, 3), \dots (1, n)$. (Hint: Use $(1, j)(1, i)(1, j) = (i, j)$.)
(b) Prove that S_n is generated by $(1, 2)$, $(2, 3), \dots (n - 1, n)$.
(c) Prove that S_n is generated by the two cycles $(1, 2)$ and $(1, 2, \dots, n)$. (Hint: Use $(1, 2, \dots, n)(i - 1, i)(1, 2, \dots, n)^{-1} = (i, i + 1)$.)

7. Show that A_n $(n \geq 4)$ and S_n $(n \geq 3)$ have trivial centers.

8. Show that $Aut(S_3)$ consists of only inner automorphisms and is isomorphic to S_3.

9. Let $N = \{e, (12)(34), (13)(24), (14)(23)\} \subset S_4$. Show that S_4/N is isomorphic to S_3.

10. Prove that the canonical homomorphism $\pi : S_4 \to S_4/N$ has a right inverse, i.e., a homomorphism $f : S_4/N \to S_4$ such that the composition $\pi \circ f$ is the identity of S_4/N.