HOMEWORK 2

1. Show that $(a, b) = (a, c)$ if $b \equiv c\ (mod\ a)$.

2. Show that $(a + b)^p \equiv a^p + b^p\ (mod\ p)$ if p is prime.

3. Find all classes $X \in \mathbb{Z}/300\mathbb{Z}$ such that:
(i) $[7] \cdot X = [2]$,
(ii) $[120] \cdot X = [80]$,
(iii) $[9] \cdot X = [48]$.

4. Find all $m \in \mathbb{N}$ such that $[5] \cdot [17] = [3] \cdot [4]$ in $\mathbb{Z}/m\mathbb{Z}$.

5. Show that every nonzero class $[a] \in \mathbb{Z}/13\mathbb{Z}$ is equal to $[2]^i$ for some i.

6. Find the (multiplicative) inverse of $[100]$ in $\mathbb{Z}/173\mathbb{Z}$.

7. Solve $X^2 = [5]$ in $\mathbb{Z}/11\mathbb{Z}$.

8. Find all $k \in \mathbb{N}$ such that $[2]^k = [1]$ in $\mathbb{Z}/17\mathbb{Z}$.

9. Let X be the set of all pairs (a, b), $a, b \in \mathbb{R}$ such that $a^2 + b^2 > 0$. We write $(a, b) \sim (c, d)$ if $ad = bc$. Show that $\sim$ is an equivalence relation and determine all equivalence classes.

10. Prove that for every odd integer a, $a^{2^{n-2}} \equiv 1\ (mod\ 2^n)$ for any $n \geq 3$. (Hint: Use Induction 1 on n)