**Homework 2** (Due: We, 10/24)

**Problem 1:** Prove the following two analytic facts (they were used in class).

a) Let $f\colon \mathbb{D} \to \Omega := f(\mathbb{D}) \subseteq \mathbb{C}$ be a conformal map with $f(0) = 0$ and $f'(0) = 1$. Then $g = f^{-1}\colon \Omega \to \mathbb{D}$ is defined on $B(0, 1/4) \subseteq \Omega$.

   Show that if $\epsilon > 0$, then $g$ is $L$-Lipschitz on $B_\epsilon := B(0, 1/4 - \epsilon)$ (i.e., $|g(w_1) - g(w_2)| \leq L|w_1 - w_2|$ for $w_1, w_2 \in B_\epsilon$) with $L = L(\epsilon)$ only depending on $\epsilon$.

b) Suppose that the sequence $\{f_n\}$ forms a normal family of holomorphic functions on a region $\Omega \subseteq \mathbb{C}$. Show that $\{f_n\}$ converges to a function $f\colon \Omega \to \mathbb{C}$ locally uniformly on $\Omega$ if and only if we have $g = h$ whenever $g$ and $h$ are two subsequential limits of $\{f_n\}$ with respect to locally uniform convergence on $\Omega$.

**Problem 2:** Let $w_0 \in \mathbb{C}$. Suppose that $\Omega$ and $\Omega_n$ for $n \in \mathbb{N}$ are regions in $\mathbb{C}$ containing $w_0$. Show that $\Omega_n \to \Omega$ in the sense of kernel convergence with respect to $w_0$ if and only if the following two conditions are true:

(i) if $K \subseteq \Omega$ is compact, then $K_n \subseteq \Omega_n$ for sufficiently large $n$.

(ii) if $w \in \partial\Omega$ is arbitrary, then there exist points $w_n \in \partial\Omega_n$ for large $n$ such that $w_n \to w$ as $n \to \infty$.