Homework 1 (Due: We, 10/17)

Problem 1:

a) Show that a compact set $K \subseteq \mathbb{C}$ is locally connected if and only if for each $\epsilon > 0$ the set K can be written as a finite union $K = K_1 \cup \cdots \cup K_n$ of compact connected sets $K_i \subseteq K$ with $\operatorname{diam}(K_i) < \epsilon$.

b) Show that if $K \subseteq \mathbb{C}$ is compact and locally connected, and $f \colon K \to \mathbb{C}$ is continuous, then $f(K)$ is also locally connected.

Problem 2: Show that a conformal map $f \colon \mathbb{D} \to \Omega$ onto a simply connected region $\Omega \subseteq \widehat{\mathbb{C}}$ cannot collapse a non-empty open subset of $\partial\mathbb{D}$ to a point in $\widehat{\mathbb{C}}$; more precisely, show that if $w \in \widehat{\mathbb{C}}$ is arbitrary, and $A \subseteq \partial\mathbb{D}$ is the set of all $a \in \partial\mathbb{D}$ such that $f(z_n) \to w$ whenever $\{z_n\}$ is a sequence in $\mathbb{D}$ with $z_n \to a$, then A has empty interior in $\partial\mathbb{D}$.