Homework 1 (Due: We, 10/17)

Problem 1:

a) Show that a compact set $K \subseteq \mathbb{C}$ is locally connected if and only if for each $\epsilon > 0$ the set $K$ can be written as a finite union $K = K_1 \cup \cdots \cup K_n$ of compact connected sets $K_i \subseteq K$ with $\text{diam}(K_i) < \epsilon$.

b) Show that if $K \subseteq \mathbb{C}$ is compact and locally connected, and $f: K \rightarrow \mathbb{C}$ is continuous, then $f(K)$ is also locally connected.

Problem 2: Show that a conformal map $f: \mathbb{D} \rightarrow \Omega$ onto a simply connected region $\Omega \subseteq \mathbb{C}$ cannot collapse a non-empty open subset of $\partial \mathbb{D}$ to a point in $\hat{\mathbb{C}}$; more precisely, show that if $w \in \hat{\mathbb{C}}$ is arbitrary, and $A \subseteq \partial \mathbb{D}$ is the set of all $a \in \partial \mathbb{D}$ such that $f(z_n) \rightarrow w$ whenever $\{z_n\}$ is a sequence in $\mathbb{D}$ with $z_n \rightarrow a$, then $A$ has empty interior in $\partial \mathbb{D}$. 