Homework 2 (due: Fr, 3/17)

Problem 1: Problem 6.2.4 in Durrett.

Problem 2: Problem 6.2.6 in Durrett.

Problem 3: Problem 6.2.8 in Durrett.

Problem 4: Let $(\Omega, \mathcal{F}, \mathbb{P})$ be an underlying probability space, $(S, \mathcal{S})$ be a measurable space, X and Y be S-valued random variables, $\mathcal{A} \subseteq \mathcal{F}$ be a σ-algebra, and $\varphi\colon S \times S \to \mathbb{R}$ be a bounded measurable function.

Suppose that X is $\mathcal{A}$-measurable (i.e., $X^{-1}(M) \in \mathcal{A}$ for each $M \in \mathcal{S}$) and that $\mathcal{A}$ and $\sigma(Y)$ are independent. Show that then

$$\mathbb{E}(\varphi(X, Y)|\mathcal{A}) = g(X),$$

where $g\colon S \to \mathbb{R}$ is defined as $g(x) = \mathbb{E}(\varphi(x, Y))$ for $x \in S$.

This is Lemma 6.2.1 is Durett's book (with slightly different notation), but the proof is a bit sloppy. Carefully justify all steps and in particular the measurability of the relevant functions (for example, why is g measurable?).