Homework 4 (due: Fr, 1/30)

Problem 1: Let Y and $X_1, \ldots, X_n$ with $n \in \mathbb{N}$ be random variables. Show that Y is $\sigma(X_1, \ldots, X_n)$-measurable if and only if there exists a Borel function $f\colon \mathbb{R}^n \to \mathbb{R}$ such that $Y = f(X_1, \ldots, X_n)$.

Problem 2: Exercise 5.1.10. Hint: Condition on events is a suitable partition of the underlying probability space. Carefully justify the steps in your computation.