Homework 2 (due: Fr, 1/16)

Problem 1: Let N_t, $t \in [0, \infty)$, be a Poisson process with rate $\lambda \geq 0$ on some underlying probability space $(\Omega, \mathcal{F}, \mathbb{P})$. For $t \in [0, \infty)$ define

$$N_{t^-} := \lim_{s \to t^-} N_s.$$

So N_{t^-} is the left-hand limit of the process at time t (which exists almost surely). Note that $N_t - N_{t^-} \geq 1$ precisely if the process jumps at time t. Show that almost surely the process has no "multiple jumps", i.e.,

$$\mathbb{P}(\{\omega \in \Omega : \text{there exists } t \in [0, \infty) \text{ s.t. } N_t(\omega) - N_{t^-}(\omega) \geq 2\}) = 0.$$

Problem 2: Exercise 3.6.12 on p. 157 in Durrett.