Homework 1 (due: Fr, 1/9)

Problem 1:

a) Suppose X and Y are independent random variables with $X \sim \text{Poisson}(\alpha)$ and $Y \sim \text{Poisson}(\beta)$, where $\alpha, \beta \geq 0$. Show that then

$$Z = X + Y \sim \text{Poisson}(\alpha + \beta).$$

b) Let Z be a Poisson random variable, and X and Y be two independent $\mathbb{N}_0$-valued random variables such that $Z = X + Y$. Show that then X and Y are also Poisson random variables.

Hint: Show that there exists an entire function F such that $\varphi_X(t) = F(e^{it})$ for $t \in \mathbb{R}$ and that F has *exponential type*, i.e., there exists a constant $C \geq 0$ such that $|F(z)| \leq C \exp\{C|z|\}$ for $z \in \mathbb{C}$.

Problem 2:

a) Let X be a random variable with characteristic function φ_X. Show that for $n \in \mathbb{N}$ we have $\varphi_X \in C^{2n}(\mathbb{R})$ if and only if X has a finite moment of order $2n$, i.e., $\mathbb{E}(X^{2n}) < \infty$.

b) Why is a corresponding result not true for moments of odd order?