Homework 6 (due: Fr, Nov. 3)

(1) The last price of Wells Fargo (Ticker: WFC) on Thursday, 10/26/17, was \$55.62. Options with expiration 11/17/17 had following last prices:

call	strike	put
1.11	55	0.78
0.85	55.5	1.04
0.63	56	1.33

The spreads of the options were fairly low in the 0.04–0.06 range. The indicated price for the options is the average of bid and ask, which gives a good approximation for a realistic price. The call-put parity (neglecting interest rates) requires the (approximate) relation $K + c = S_0 + p$ between the current stock price S_0, the strike price K, the price c of the call, and the price p of the put.

(a) Compute $S_0 + p - K - c$ for each of the three strike prices.

(b) The computations in (a) show that the call-put parity seems to be violated. Explain the reason for this!

(2) (a) A *call ratio spread* is implemented by buying a call O_1 and selling a larger number of calls O_2 at a higher strike price (with the same underlying and expiration). Suppose we buy one option O_1 with strike K_1 for the price c_1 and sell two options O_2 with strike $K_2 > K_1$ for the price c_2 each. Analyze the profit of this strategy depending on the price S_T of the stock at expiration T and represent this in a diagram.

(b) We consider a bull spread involving two call options O_1 and O_2 (with the same underlying and expiration) with strike K_1 and K_2, respectively, where $K_1 < K_2$. Devise a strategy based on put options that gives the same payoff as the bull spread.

(c) Devise a strategy based on options that leads to the same payoff as short selling the stock.

(3) (a) Suppose X_1 and X_2 are continuous random variables with probability density functions p_1 and p_2, respectively. Suppose that X_1 and X_2 are independent. Express the probability density function q of $Y = X_1 + X_2$ in terms of an integral involving the functions p_1 and p_2.

(b) Suppose that in (a) the the random X_i is a normal random variable with mean μ_i and standard deviation σ_i for $i = 1, 2$. Use the derived formula in (a) to find the probability density function of $Y = X_1 + X_2$ and to show that Y is also a normal random variable. What are the mean and the standard deviation of Y?

(c) Suppose $X_1, \ldots, X_n$ are independent normal random variables with the same mean μ and standard deviation σ. Show that $S = X_1 + \cdots + X_n$ is also a normal random variable and find its mean and standard deviation.

(4) The purpose of Problems 4 is to develop some of the theoretical foundations for describing random stock price movements (cf. HW3, Prob. 4).

(a) Let Z_1 be a random variable representing a random quantity at time $t = 1$ (such as the random change of the price of an asset from time 0 to time 1). Suppose that Z_1 has mean μ and standard deviation σ. We divide the time $t = 1$ into a large number n of time intervals and suppose that each of the time intervals contributes independently in the same way to Z_1. More precisely, we assume that Z_1 is approximately given by a sum

$$X_1 + \cdots + X_n,$$

where $X_1, \ldots, X_n$ are i.i.d. random variables.

What do we have to assume about the mean a_n and the standard deviation b_n of X_i, $i = 1, \ldots, n$, so that this can be valid?

(b) Let $n \to \infty$ in (a). What can we then say about the distribution of the random variable Z_1?

(c) Suppose we use the same model as in (a) and (b) for an arbitrary time $t \geq 0$. Then we divide t into $k = nt$ time intervals of length $1/n$ and assume that Z_t is approximately given by the sum

$$X_1 + \cdots + X_k.$$

Letting $n \to \infty$, what can we say about the distribution of Z_t? In particular, what are the mean μ_t and the standard deviation σ_t of Z_t in terms of μ, σ, and t? What is Z_0?