Homework 5 (due: Fr, Oct. 27)

(1) A bond has a par value of \$100, a coupon of 4% with semiannual payments, and a maturity of 2 years. It current market price is \$98. The purpose of this problem is to compute the yield x of the bond (with continuous compounding).

(a) Find the relevant equation for x and write it in the form $f(x) = 0$.

(b) A first approximation for x is $x_0 = .04$ (the coupon of the bond). Find the next three approximations x_1, x_2, x_3 for x using Newton's approximation method based on the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

(c) What is the value of x up to a basis point (=a hundredth of a percent)?

(2) (a) Suppose a bond has a current market price B, a yield y, and generates a cash payment c_i at time t_i (measured in years) for $i = 1, \dots, n$. Then the quantity

$$D = \frac{1}{B} \sum_{i=1}^{n} t_i c_i e^{-y t_i}$$

is called the *duration* of the bond (this is a technical term not to be confused with the maturity of the bond). What is the duration D of the bond in (1)?

(b) Show that a small change ΔB of the price of the bond is related to a small change Δy of its yield by the formula

$$\Delta B = -BD\Delta y.$$

(c) Suppose the price of the bond in (1) drops to \$97.50. What is the approximate yield of the bond at this new price based on (a) and (b)?

(3) You are interested in investing a larger sum of money in bonds with short maturity. You have two bonds B_1 and B_2 under consideration. Both have a par value of \$100 and a maturity of one year with semiannual interest payments. The bond B_1 has a coupon of 6% and a current market price of \$99, and the bond B_2 a coupon of 1% and a market price of \$94.10. Which bond is the better investment? Justify your answer!

(4) We consider three European call options with the same underlying and the same time to expiration. Suppose the strikes of the options are $K_1 < K_2 < K_3$ and their prices are c_1, c_2, c_2, respectively.

(a) Suppose that $K_2 - K_1 = K_3 - K_2$. Show that then $c_2 \leq \frac{1}{2}(c_1 + c_3)$. Hint: Create two portfolios and compare their values when the options expire.

(b) Find a similar bound for c_2 in terms of c_1 and c_3 without the assumption that $K_2 - K_1 = K_3 - K_2$.