Homework 4 (due: Fr, Oct. 20)

(1) Find the prices for shares of Microsoft Corp. and of call options on this stock with 12/15/17 expiration (use a day between 10/13 and 10/19 after market close). Plot (i) the last price of the options for strikes between \$70 and \$85 in steps of \$2.50 and (ii) their time values. Can you formulate a general rule about the time values of the options from the data?

(2) (a) A bank quotes an interest rate of 8% with quarterly compounding. What are the equivalent rates with (i) annual compounding and with (ii) continuous compounding?

(b) An investor receives \$1,100 in one year for an investment of \$1,000. What is the percentage return per year with (i) semiannual compounding and with (ii) monthly compounding?

(c) A deposit pays 12% per year with continuous compounding, but interest is actually paid quarterly. How much interest will be paid each quarter on a \$10,000 deposit?

(3) (a) You play a game where you have to pay an upfront cost C to enter the game. You cannot lose any more money and your gain depends on the value of a real-valued random variable Y. Namely, if Y is negative, then you do not gain anything. If Y is positive, then you gain the amount Y.

What is the *fair value* of C, i.e., the value C where on average (when you play the game repeatedly, one game does not influence the following games, and you do not care about the risk involved) you neither gain nor lose money in the long run? Hint: Use the expectation of a random variable related to Y.

(b) Suppose the game is based on a coin flip, where you gain a dollar if heads comes up and you gain nothing if tails comes up. What is the fair value c to enter this game?

(c) Suppose you consider a call on a stock with strike K and time to expiration T. We consider the stock price at time T as a random variable S_T. What is the fair value c of the call in terms of an expectation of a suitable random variable if we use the approach in (a) (and ignore interest rates and discounting)?

(4) (a) We use the simplistic probabilistic model for the price S_n of a stock in n days from now that was discussed in HW3, Prob. 4. Then for large n we have approximately

$$S_n = S_0 + a_n N,$$

where S_0 is the current stock price and N is a standard normal random variable. Express a_n in terms of σ and n.

(b) Suppose c is the fair value of a call at the money, i.e., for its strike K we have $K = S_0$. We assume its time to expiration T is large. Use the method from 3(c) to express the fair price c of the call in terms of S_0, T, and σ. Hint: Use HW3, Prob. 2(e).

How is this related to HW3, Prob. 2?