Homework 2 (due: Fr, Oct. 6)

(1) (a) Find the option chain for Facebook (ticker symbol: FB) on one of the days Oct. 2–Oct. 5 after markets close (1pm LA time). Make a table with the following information: last price for call/put options with strikes 160, 170, 180 and expirations Oct. 20, 2017 and Jan. 19, 2018 (a total of 12 values).

(b) Explain the terms *volume* and *open interest* that appear in the option chain.

(2) (a) Review the *central limit theorem* from some source (probability textbook, internet, etc.).

(b) Suppose X is a random variable that only takes the values $X = -1$ and $X = 1$ with equal ($= 0.5$) probability. What are the expected value μ_0 and the standard deviation σ_0 of X?

(c) Suppose $X_1, X_2, X_3, \dots$ are random variables that are independent and have the same distribution as the random variable X in (b). Suppose n is a natural number. What is the mean μ_n and what is the standard deviation σ_n of

$$Z_n = X_1 + \cdots + X_n?$$

(d) Suppose n in (c) is very large. Then the central limit theorem suggests that Z_n has a distribution close to the distribution of $a_n + b_n N$, where N is a standard normal random variable. What are a_n and b_n here?

(3) You are collecting money for a charity and want to raise \$10,000. For this purpose you send out letters to randomly chosen people asking for a donation of \$10. From past experience you know that 10% of the people asked will donate \$10 (and 90% will give nothing). The purpose of this problem is to find a number n such that if n letters are sent out, you will collect the desired amount of \$10,000 with about 95% certainty. This number n should be as small as possible, because you do not want to send out more letters than necessary.

(a) Define a (discrete) random variable X that models the amount received from one donor.

(b) Suppose that the potential donors receiving your letters do not influence each other in their decision whether to make the \$10 donation or not. Find a formula for the total amount S_n raised if n letters are sent out. To find this formula, use the concept of i.i.d. (independent and identically distributed) random variables and relate them to X defined in (a).

(c) Justify why for large n we can assume that S_n has a distribution closely approximated by the distribution of a normal random variable Z_n. What are the mean μ_n and the standard deviation σ_n of Z_n here?

(d) Let N be a standard normal random variable. Find a real number a_n such that

$$\text{Probabilty}(S_n < 10,000) \approx \text{Probabilty}(Z_n < 10,000) = \text{Probabilty}(N < a_n).$$

Hint: Find c_n and d_n such that $c_n N + d_n$ has the same distribution as Z_n.

(e) Let N be a standard normal random variable. Find the value a such that

$$\text{Probabilty}(N < a) = 0.05.$$

Hint: This is related to the cumulative distribution function of N. Its values can be looked up in tables or can be computed from built-in functions in standard programs for statistical computation.

(f) What is the number n of letters that should be sent out?

(4) You play a game that is stacked in your favor as follows. A (fair) coin is flipped. If it comes up tails, then you lose the amount B that you bet, but if it comes up heads then they your money is tripled (i.e., you receive the original amount B back plus $2B$ in winnings). You start with a (large) amount M_0 of money. Let M_n be your total amount after you played the game n times. We now consider two scenarios.

(a) Each time you bet $B = 1$. Express M_n in terms of M_0 and suitable i.i.d. random variables $X_1, X_2, \dots$. Describe the distribution of X_1!

(b) In this second, more interesting scenario we fix a number $p \in [0, 1]$. Each time we bet the fraction p of the total amount that we have at this point. Express M_n again in terms of M_0 and suitable i.i.d. random variables $Y_1, Y_2, \dots$. Describe the distribution of Y_1!

(c) The game in your favor. So should you choose $p = 1$ in (b), i.e., bet *all* the money you have each time? If not, which p should we choose to optimize the long-term outcome of the game? Hint: Typically, $M_n \approx M_0 e^{an}$ for large n, where a depends on p. You want to maximize a.