Homework 1 (due: Mo, Oct. 2)

Note: Homework has to be turned in as a paper copy on the due-date before class starts. I will not accept late homework. Under exceptional circumstances I am happy to give an extension if I am asked 24 hours before the due-date.

(1) Define the terms *market capitalization*, *price/earning ratio*, and *dividend yield* for a stock.

(2) Which are the five largest US companies according to market capitalization on the stock market? Create a table for the five companies with the following information: ticker symbol, current stock price (after markets close—pick your favorite day), market capitalization, price/earning ratio, dividend yield. Indicate the source where you obtained this information.

(3) Use Hopital's rule to compute the limit $\lim_{n\to\infty}\big(1+x/n\big)^n$ for a real number x. Show work!

(4) Review Taylor series from your calculus textbook or some other source. In the following two problems you are supposed to compute the Taylor expansion for a function $f(x)$ at $x_0 = 0$. The Taylor expansion then has the form

$$f(x) = \sum_{n=0}^{\infty} a_n x^n,$$

where the a_n are explicit constants depending on the function f. Find this Taylor series (i.e., the coefficients a_n) for the function

(a) $f(x) = e^x$,

(b) $f(x) = \log(1 + x)$.

Show work!

(c) Why should we care about Taylor series and what are they good for? (You may first want to do Problem (5) before you answer this question).

(5) You invest $\$1,000$ for 10 years at a 5% yearly interest rate. After each year the interest paid is reinvested at the same rate.

(a) Represent the total amount A after ten years in the form $A = S(1 + x)^n$. What are S, x, and n here?

(b) Compute A (up to cents) using a calculator.

(c) If one writes A in the form $A = S(1+x)^n = S\exp(n\log(1+x))$ and uses the approximation $\log(1+x) \approx x$ for small x, then one obtains $A \approx Se^{nx}$. Which value for A is obtained with this approximation method?

(d) A better approximation is $\log(1+x) \approx x - \frac{1}{2}x^2$. Explain how this is related to Problem 4(b).

(e) Based on the approximation in (d) develop an approximation formula for A in terms of S, x, and n. Use it to compute A in our specific example.