Math 174E, Mathematics of Finance

Fall 2016

Midterm 1

Name:

UID:

There are five problems with a total of 50 points.

Problem 1: (a) You are considering buying a stock for its high dividend yield. The current stock price is \$50 per share with a quarterly dividend of \$0.50. What is the annual dividend yield r if we assume that dividends are reinvested in the stock at the same price per share? It is enough to write down the equation for r based on the given numbers without performing the explicit computation. (4pts)

(b) A bond has a maturity of 2 years, a coupon of 2% with semiannual payments, and a current price of \$99. Write down the equation for the yield y of the bond with continuous compounding (you do not have to solve the equation!). (4pts)

(c) At the beginning of each year you pay an amount A into a savings account. For your savings you receive an annual interest rate of $r\%$ with annual compounding. Find a simple formula (NOT involving a sum of n terms) for the total amount S_n of money saved at the end of the nth year.

Hint: To find an explicit formula for a sum of type $a + a^2 + \cdots + a^n$, multiply by $a - 1$! (4pts)

Problem 2: (a) You are considering two call options O_1 and O_2. The underlying of the options is AAPL whose current price is \$110. The expiration of both O_1 and O_2 is December 2016. The option O_1 has strike \$100 and a current price of \$14, while O_2 has strike \$105 and a current price of \$8.50 (we assume that spreads are negligible). Show that this is an arbitrage opportunity and devise a suitable strategy of how to exploit it. (4pts)

(b) With the strategy that you devised in (a) consider the following three scenarios: The price of AAPL at (or very near) expiration of the options is \$95, \$105, or \$115. What is your gain (per share of the underlying) in each of these three scenarios? (4pts)

(c) In theoretical models one often assumes that there are no arbitrage opportunities (with the idea that in liquid markets any such opportunity would quickly lead to a change of prices eliminating it). Based on this assumption, can you derive a simple general rule from the considerations in (a) and (b) about values and strike prices of options *in the money* with the same underlying and the same expiration? (4pts)

(d) Formulate a similar rule as in (c) for options *out of the money* (with the same underlying and the same expiration)! Justify your answer! (4pts)

Problem 3: Explain the following concepts for futures contracts:

(a) initial margin, (3 pts)

(b) maintenance margin, (3 pts)

(c) margin call. (3 pts)

Problem 4: The *central limit theorem* in probability and statistics makes a statement about expressions of the form

$$(*) \qquad \frac{X_1 + \cdots + X_n - n\mu}{\sigma\sqrt{n}},$$

where $X_1, X_2, \ldots$ are random variables satisfying certain conditions. What are the hypotheses in the central limit theorem and what is the conclusion in the central limit theorem for the expression (*)? (6 pts)

Problem 5: We want to numerically solve the equation $x^3 = 2$ by using Newton's method.

(a) Find the formula for the approximation x_{n+1} in terms of x_n that applies here! (5 pts)

(b) If we here use $x_0 = 1$ as our first approximation, what is x_1? (2 pts)