

MATH 33A PRACTICE MIDTERM

Problem 1. Consider the basis $\mathcal{B} = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ of \mathbb{R}^3 . Let T be a linear transformation such that in this basis, the matrix A of T is

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}.$$

Find the matrix of T in the standard basis.

Problem 2. Let A be an $n \times n$ matrix. What can you say about the determinant of A if

- a) A is nilpotent?
- b) A is orthogonal?

Problem 3. Find a basis for the image and kernel of the following matrix:

$$B = \begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ 3 & 0 & 2 & 1 & 8 \\ 4 & 2 & 0 & 2 & 6 \\ 2 & -2 & 3 & 1 & 9 \end{bmatrix}.$$

Problem 4. Find the QR factorization of

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 3 \\ 2 & 1 & 0 \end{bmatrix}.$$

Problem 5. a) Find the line(s) that best fit the points $(-1, 1)$, $(0, 0)$, and $(1, 2)$.

- b) Find a quadratic polynomial that hits all three of the above points.

Problem 6. Consider the matrices

$$A = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 1 & 0 & -2 & 0 \\ 3 & -1 & 1 & -2 \\ 4 & -3 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & 3 & 2 \\ 3 & 0 & 1 & -2 \\ 1 & -1 & 4 & 3 \\ 2 & 2 & -1 & 1 \end{bmatrix}.$$

Find $\det(A)$, $\det(B)$, and $\det(AB)$.