Errata and Comments to
"Asymptotic Differential Algebra and
Model Theory of Transseries"

August 31, 2020

The changes below apply to the edition published by Princeton University Press, and are already reflected in the versions posted on the arXiv and on our personal web pages. We thank Allen Gehret for pointing out most of the errors left in that edition. Linguistic slips like missing commas or articles are not listed below unless they might mislead.

Acknowledgments:

The date of September 2015 on p. xiv indicates when the manuscript was first submitted to Princeton University Press. The published version incorporates some changes and additions made since then.

Dramatis Personae:

In the item for "ω-free" under the heading "Asymptotic Fields", $f - \omega(g^{\dagger\dagger}) \succcurlyeq g^{\dagger}$ should be $f - \omega(g^{\dagger\dagger}) \succcurlyeq (g^{\dagger})^2$.

Chapter 1:

(1) The first sentence of the subsection **Irreducibility** in Section 1.1 should be: *Let X and Y be topological spaces.*

(2) In the subsection **Localization of modules** in Section 1.4 the formula for addition should have $s_2 x_1 + s_1 x_2$ in the numerator.

(3) In the subsection **Tensor products** in Section 1.7, the H in the 4th line should be a B.

(4) In the subsection **Rational rank** in Section 1.7, in the line following the display: $\mathbb{Q} \otimes_{\mathbb{Z}} N$ should be $\mathbb{Q} \otimes_{\mathbb{Z}} M$.

(5) In the 4th line of the proof of Lemma 1.8.12, the second ":=" should be "=".

(6) In the 6th line of the proof of Lemma 1.8.13, "$(a, b) \to$" should be "$(a, b) \mapsto$".

(7) In Corollary 1.9.6 one should add the assumption that L is separably generated over K, that is, L is separably algebraic over an intermediate field $K(B)$ with $B \subseteq L$ algebraically independent over K. This assumption is satisfied if $\operatorname{char} K = 0$. Corollary 1.9.7 is still correct as stated, but its proof requires for positive characteristic a variant of Corollary 1.9.6, namely: *L is separably algebraic over K iff every derivation on L extending the trivial derivation on K is trivial.* (This variant with a proof, as in [249, pp. 370–371] is now

included in the arXiv version.) Lemma 1.9.8 should be restricted to the case char $K = 0$.

Chapter 2:

(1) In the fourth paragraph of Section 2.3, replace "valued subgroup of (G, S, v)" by "valued subgroup of (G', S', v')".

Chapter 3:

(1) In the second sentence of the proof of Proposition 3.1.21, one can omit "with $\mathfrak{q} \cap A = \mathfrak{q}' \cap A = \mathfrak{m}$" since this condition is automatically satisfied.
(2) F.-V. Kuhlmann pointed out that in the "Notes and comments" to Section 3.2 we misattribute Corollary 3.2.26 to Krull [229]. An early source for a result of this kind is Theorem 11 in O. Schilling's book,

> *The Theory of Valuations,* Mathematical Surveys, no. 4, American Mathematical Society, New York, 1950.

This book refers for this theorem to I. Kaplansky's unpublished Ph.D. thesis
> *Maximal Fields with Valuations,* Harvard University, 1941.

(3) Replace "theorem" by "proposition" in the sentence following the statement of Proposition 3.4.22.
(4) Marcus Tressl alerted us to an error in the proof of Theorem 3.6.11: replace the condition $\boldsymbol{K} \preccurlyeq \boldsymbol{F}$ in the first sentence of the proof by $\boldsymbol{K} \subseteq \boldsymbol{F}$, so that Zorn's lemma can be applied as indicated in the next sentence.
(5) Right after Lemma 3.7.6, replace
"open ball of the form $\{y : v(y - f) > vf\}$ where $f \in K^\times$" by
"open ball of the form $\{y : v(y - f) > vg\}$ where $f, g \in K^\times$, $f \succcurlyeq g$".

Chapter 4:

(1) In the first sentence of the proof of 4.1.10, omit *be*.
(2) The last three sentences of the proof of 4.6.12 can be shortened to: *Then by Lemma 1.3.10, a is algebraic over K, so a is algebraic over C by Lemma 4.1.2.*

Chapter 5:

(1) In line 5 of Section 5.5, replace $K[\partial]$ by $R[\partial]$.
(2) In Lemma 5.7.3, replace "$\mathbb{Q}[\phi, \ldots, \partial^n(\phi)] = \mathbb{Q}[\phi, \ldots, \delta^n(\phi)]$" by
"$\mathbb{Q}[\phi, \ldots, \partial^n(\phi), \phi^{-1}] = \mathbb{Q}[\phi, \ldots, \delta^n(\phi), \phi^{-1}]$"

Chapter 6:

(1) In the second to last line of the proof of Lemma 6.1.9, replace C by D_0.
(2) In the second line before the first display in the proof of Theorem 6.3.2 there is a misplaced parenthesis in $K[Y, \ldots, Y^{(r-1)}]$.
(3) In the last line of the proof of Lemma 6.6.5, replace (ii) by (iii).

Chapter 7:

(1) In the third line of the proof of Proposition 7.5.6, replace E by $E^\times$.

Chapter 8:

(1) In the proof of Corollary 8.3.2, $(E, \Gamma, \boldsymbol{k}_E)$ should be $(E, \boldsymbol{k}_E, \Gamma)$.
(2) A few lines before Corollary 8.3.3, $\theta_v(v_1, \ldots, v_k, y)$ should be $\theta_v(v_1, \ldots, v_k, z)$.
(3) In the proof of Proposition 8.4.12, third line from the bottom, "$\Gamma_{K_3} = \Gamma_{K_3}$" should be "$\Gamma_{K_2} = \Gamma_{K_3}$".

Chapter 9:

(1) Two lines before Corollary 9.1.10, (3) should be (2).
(2) Replace "Lemma" in the last line of the proof of Lemma 9.2.17 by "Corollary".
(3) The correction following Lemma 3.7.6 leads to a corresponding correction in describing the condition $z \in G_i$ when $s_i \neq 0$, in the proof of Lemma 9.7.3.
(4) Verifying (AC3) in proof of Lemma 9.8.2 can be shortened using

$$\max\left\{\psi^\alpha(\gamma + k\alpha) : \ \gamma \in \Gamma, \ k \in \mathbb{Z}, \ \gamma + k\alpha \neq 0\right\} \ = \ \beta - \alpha.$$

(5) In proof of Lemma 9.9.3, insert right after "v-slow on the right" the phrase ", where v is the standard valuation of Γ".

Chapter 10:

(1) In the last sentence of the third paragraph in the "Notes and comments" to Section 10.6, "not not" should be "not".

Chapter 11:

(1) In the last display before Lemma 11.1.4, the expression $\{\gamma : \ \gamma < (\Gamma^>)'\}$ should be replaced by $\{\gamma \in \Gamma : \ \gamma < (\Gamma^>)'\}$.
(2) In Lemma 11.2.3(ii), complete to "nmul $P = $ nmul P_{+a}" at the end.
(3) In proof of Lemma 11.6.3, replace $v(s-a^\dagger) \in (\Gamma_F^>)'$ by $v(s-a^\dagger) \in (\Gamma_F^>)' \cup \{\infty\}$.
(4) In last sentence of proof of Lemma 11.6.14, replace $\sim sf$ by $\sim -sf$.
(5) In proof of Proposition 11.6.17, end of the fourth paragraph, replace λ by $\boldsymbol{\lambda}$.
(6) In second part of Lemma 11.8.5, omit the assumption that K has asymptotic integration and replace $=$ at end of proof by $\subseteq$.
(7) Omit the proof of Corollary 11.8.13; it has an erroneous forward reference.

Chapter 14:

(1) In the line following the statement of Theorem 14.0.1, it would be better to refer to Corollary 11.7.13 than to Corollary 11.7.10.
(2) In the third line of the proof of Lemma 14.1.8, replace K by $K^\times$.
(3) In the last line of the last display preceding Proposition 14.2.18, replace Y'' by $Y''Y$.
(4) In Lemma 14.3.2 (iii), replace *at newton position* by *in newton position*.
(5) In line 6 of the proof of Lemma 14.3.2, after "nmul $P_{+b} = $ nmul $P_{+a} = 1$" add "by Lemma 11.2.3" (referring to the addition to 11.2.3(ii) made above).
(6) After Corollary 14.5.11, replace *In Section 16.1* by *In Section 16.2*.

Appendix A:

(1) In the sixth line before the subsection "Representing $\mathbb{T}$ ..." on p. 719, replace $[v(\ell_{n-1})$ by $[v(\ell_{n-1})]$.

Appendix B:

(1) Add to the "Notes and comments" of Section B.12 that Corollaries B.12.9 and B.12.11, with different proofs, are from:

> A. H. Lightstone, A. Robinson, *On the representation of Herbrand functions in algebraically closed fields,* J. Symb. Logic **22** (1957), 187–204.