

Problem Set 6
Due Friday, Dec. 3.

Formal Logic

Math 430, Fall 2004

1. (Extra credit.) Let $S = \{<\}$, where $<$ is a 2-place relation symbol, and let $\mathcal{Q} = (\mathbb{Q}, <^{\mathcal{Q}})$, where $<^{\mathcal{Q}}$ is the usual ordering on \mathbb{Q} . Show that the set

$$\Phi = \{\varphi : \varphi \text{ is an } S\text{-sentence and } \mathcal{Q} \models \varphi\}$$

is consistent.

2. (Extra credit.) Suppose that Φ is a maximally consistent set of S -formulas, where S is some symbol set. Show that for all S -formulas ψ and θ : if $(\neg\psi \vee \theta) \in \Phi$ and $\psi \in \Phi$, then $\theta \in \Phi$.
3. (Extra credit.) Give an example of a symbol set S and a consistent set of S -formulas that is *not* maximally consistent.
4. (Extra credit.) Suppose that Φ is a maximally consistent set of S -formulas. We define the equivalence relation \sim on the set \mathcal{T}_S of all S -terms as usual: $t \sim t' \iff t = t' \in \Phi$. We also put $\bar{t} = \{t' \in \mathcal{T}_S : t \sim t'\}$. Show:

$$t \sim t' \iff \bar{t} = \bar{t}' \iff \bar{t} \cap \bar{t}' \neq \emptyset.$$