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Small profinite structures and their generalizations

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A profinite structure in the sense of Newelski is a pair $(X, Aut^*(X))$ consisting of a profinite topological space X and a closed subgroup $Aut^*(X)$ (called the structural group) of the group of all homeomorphisms of X respecting the inverse system defining X. We say that a profinite structure $(X, Aut^*(X))$ is small if for every natural number $n > 0$, there are only countably many orbits on X^n under the action of the structural group. In small profinite structures Newelski introduced a topological notion of independence, which has similar properties to those of forking independence in stable theories, and developed a counterpart of geometric stability theory in this context.

I will present this notion of independence and explain why smallness plays an important role here. I will also give some examples and results concerning small profinite groups regarded as profinite structures.

Then I will talk about my recent ideas concerning generalizations of small profinite structures to the case of: 1) non-small profinite structures; 2) 'compact structures' (i.e. X is a compact metric space and $Aut^*(X)$ is a compact group acting on X continuously); 3) 'Polish structures' (i.e. X is a Polish space and $Aut^*(X)$ is a Polish group acting on X continuously).