

UIC Model Theory Seminar, April 3, 2006

Elementary socles and radicals

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The *elementary socle* of a module is the sum of all the minimal non-zero first-order definable subgroups of that module. Dually the *elementary radical* of a module is the intersection of all the maximal proper first-order definable subgroups of that module. These concepts were first introduced by Ivo Herzog in his thesis.

If an indecomposable module has the descending chain condition on definable subgroups, the elementary socle is non trivial and is a definably closed submodule. Furthermore, the definition of elementary socle naturally extends to an ascending series of definably closed submodules whose union is the whole module. Dually, if an indecomposable module is pure-injective and has the ascending chain condition on definable subgroups, the elementary radical is a submodule, and the definition of the elementary radical may be extended to a descending series of submodules whose intersection is 0.

The definitions and some of the properties generalize in natural ways to arbitrary (indecomposable) pure-injective modules.

Mike Prest introduced a notion of duality between certain first order formulas in the languages of left modules and right modules which Herzog extended to a duality of categories. This duality makes modules of the kind described above correspond; and their internal structure is shown to be similar by means of this duality.

This is a preliminary report on work in progress.