A characterization of first-order topological properties of planar spatial databases

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A spatial query is a first-order sentence with predicates that range over semi-algebraic relations (or, more generally, definable relations in some o-minimal expansion of the real field). A query is topological if (informally) it depends only on the shape of a region. Our work concerns characterizing which first-order sentences are topological. One cannot decide whether a query is topological, but one can hope to give effective syntactic criteria capturing exactly the topological queries. Our main result is a language capturing topological queries in the case where the relation symbol is restricted to be a closed region in the plane. I’ll overview the proof, which combines some (simple, old) results about queries over o-minimal structures, along with some (pretty simple, but new) automata theory. The main tool from automata theory is a characterization of which regular languages are first-order definable relative to a modulus class. This is joint work with Christoph Loeding, Jan van den Bussche, and Thomas Wilke.