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Hilbert's Tenth Problem for function fields of positive characteristic

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Hilbert's Tenth Problem in its original form was to find an algorithm to decide, given a polynomial equation $f(x_1, \ldots, x_n) = 0$ with coefficients in the ring $\mathbf{Z}$ of integers, whether it has a solution with $x_1, \ldots, x_n \in \mathbf{Z}$. Matiyasevich proved that no such algorithm exists, i.e. Hilbert's Tenth Problem is undecidable. Since then, analogues of this problem have been studied by asking the same question for polynomial equations with coefficients and solutions in other commutative rings.

Let k be the function field of a curve over a finite field, and let v be a non-trivial discrete valuation on k with valuation ring R_v. We will give a new proof of the known result that R_v is diophantine over k, and we will show how this can be used to prove that Hilbert's Tenth Problem for k is undecidable.