An expansion of the real field is said to be o-minimal if every definable set has finitely many connected components. Such structures are a natural setting for studying “tame” objects of real-analytic geometry such as non-oscillatory trajectories of real-analytic planar vector fields. It turns out that even some infinitely spiralling trajectories of such vector fields have a reasonably well-behaved model theory; this motivates the notion of d-minimality, a generalization of o-minimality that allows for some definable sets to have infinitely many connected components. The following trichotomy illustrates why we are interested in this notion.

Let $U \subseteq \mathbb{R}^2$ be open and $F: U \to \mathbb{R}^2$ be real analytic such that the origin $0$ is an elementary singularity of $F$ (i.e., $f^{-1}(0) = \{0\}$ and the linear part of $F$ at $0$ has a nonzero eigenvalue). Let $g: (a, b) \to \mathbb{R}^2$ be a solution to $y' = F(y)$ such that $g(t) \to 0$ as $t \to a^\pm$. Then, after possibly shrinking $b$, exactly one of the following holds for the expansion $M$ of the real field by the curve $g((a, b))$:

(a) $M$ is o-minimal;
(b) $M$ is d-minimal and not o-minimal;
(c) $M$ defines the set of all integers.

I shall outline the proof of this trichotomy, and indicate how it might be generalized.