UIC Model Theory Seminar, April 13, 2004

Expansions of o-minimal structures by trajectories of definable planar vector fields

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An expansion of the real field is said to be o-minimal if every definable set has finitely many connected components. Such structures are a natural setting for studying "tame" objects of real-analytic geometry such as non-oscillatory trajectories of real-analytic planar vector fields. It turns out that even some infinitely spiralling trajectories of such vector fields have a reasonably well-behaved model theory; this motivates the notion of d-minimality, a generalization of o-minimality that allows for some definable sets to have infinitely many connected components. The following trichotomy illustrates why we are interested in this notion.

Let $U \subseteq \mathbb{R}^2$ be open and $F\colon U \to \mathbb{R}^2$ be real analytic such that the origin 0 is an elementary singularity of F (i.e., $f^{-1}(0) = \{0\}$ and the linear part of F at 0 has a nonzero eigenvalue). Let $g\colon (a, b) \to \mathbb{R}^2$ be a solution to $y' = F(y)$ such that $g(t) \to 0$ as $t \to a^+$. Then, after possibly shrinking b, exactly one of the following holds for the expansion M of the real field by the curve $g((a, b))$:

(a) M is o-minimal;
(b) M is d-minimal and not o-minimal;
(c) M defines the set of all integers.

I shall outline the proof of this trichotomy, and indicate how it might be generalized.