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K-analytic geometry over o-minimal structures

(joint work with Sergei Starchenko)

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Let M be an o-minimal expansion of a real closed field R, and let K be the algebraic closure of R.

The notion of a K-holomorphic function in one and several variables, (for M-definable partial functions on K^n) is defined in analogy to classical complex functions. K-manifolds and K-analytic subsets are defined similarly. Interesting examples of such definable objects arise from algebraic geometry and from the theory of compact complex manifolds, including its elementary extensions.

I will discuss some of the theorems which, when interpreted over the complex numbers, yield seemingly stronger theorems than the classical ones (under the assumptions that the objects are definable in some o-minimal structure!).

Theorem 1. (A finiteness theorem) *If M is a K-manifold and A is a definable closed subset of M which is locally K-analytic then there are finitely many definable open sets covering A, on each of which A is the zero set of finitely many K-holomorphic functions.*

Theorem 2. (A strong version of Remmert's mapping theorem) *Assume that $f\colon M \to N$ is a K-holomorphic map between K-manifolds, and A is a K-analytic subset of M. If $f(A)$ is a closed subset of N then it is a K-analytic subset of N.*

Theorem 3. (Strong removal of singularities) *Assume that A is K-analytic subset of a K-manifold M, and that B is a K-analytic subset of $M - A$. Then the closure of B in M is a K-analytic subset of M.*