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There are no categorical sentences in $L(Q)$!

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A first order sentence with an infinite model has models in all cardinalities; in particular no sentence is *categorical*, has exactly one model. Sentences of $L_{\omega_1,\omega}$ may be categorical but only in cardinality $\aleph_0$. In the early 70's I asked whether a sentence of $L(Q)$ could have exactly one model of cardinality $\aleph_1$. Shelah showed the answer was no with some additional set theoretic hypotheses that he later removed. We introduce methods of getting structural properties on the models in an AEC that have cardinality λ by restricting the number of models of cardinality λ^+. And from these conditions on models of cardinality λ we show the existence of a model of power λ^+. Most strikingly, we present Shelah's proof that if a sentence of $L_{\omega_1,\omega}(Q)$ is categorical in $\aleph_1$ then it has a model of cardinality $\aleph_2$.