Math 31b : Midterm 1, Spring 2012

Professor Curran

Each problem is worth 10 points.

1. Evaluate the following limits:

(a)

$$\lim_{t \to 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t}$$

(b)

$$\lim_{x \to 0+} (\cos x)^{1/x^2}$$

2.

(a) Evaluate the indefinite integral

$$\int \frac{\ln(\ln x)}{x \ln x} dx.$$

(b) Use logarithmic differentiation to compute $f'(x)$, where

$$f(x) = \frac{\sqrt{x^3+2} \cdot (x-1)^{2/3}}{(4+x^2)^3}$$

for $x > 1$.

3. Let $f(x) = x^5 + 2x$.

(a) Show that $f(x)$ is one-to-one on $(-\infty, \infty)$.

(b) Let $g(x) = f^{-1}(x)$. Find $g'(3)$.

4. A continuous annuity with withdrawal rate $N = \$1000$/year and interest rate r is funded by an initial deposit of $P_0 = \$10000$. Let $P(t)$ be the balance of the account after t years.

(a) Write the differential equation satisfied by $P(t)$.

(b) What is the smallest value of r for which the annuity will never run out of money?

(c) If $r = 5\%$, at what time will the annuity run out of funds?

5. The number of computers infected by a certain computer virus increases at a rate proportional to the number of computers currently infected. Suppose that on January 20, 2012 there were 2^{10} computers infected with the virus, and three days later there were 2^{12} computers infected.

Let $P(t)$ be the number of computers infected t days after the virus was released (i.e. after the first computer was infected).

(a) What differential equation does $P(t)$ satisfy?

(b) Find the formula for $P(t)$.

(c) On what day was the virus released?