

Course Announcement
Model Theory of Differential Fields
Math 223M, Spring Quarter 2016
MWF 1–1:50 PM, Haines A74

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Office hours. by appointment; MS 5614.

Description. This class will be an introduction to algebraic and model-theoretic aspects of differential fields, with a particular emphasis on the Galois theory of linear differential equations. This theory, which goes back to Picard and Vessiot at the end of the 19th century, parallels the Galois theory of algebraic equations. The differential Galois group carries the structure of a linear algebraic group, hence Picard-Vessiot theory served an important motivation for developing the theory of algebraic groups. The foundations of differential algebra were laid by Ritt in the early 20th century and much clarified and extended by Kolchin, who also put Picard-Vessiot theory on a firm basis. The rise of differential algebra went hand in hand with the early development of model theory. Phenomena arising in the former often served as an inspiration for the latter. This culminated in the applications of the model theory of differential fields in diophantine geometry in the 1990s by Hrushovski, Pillay-Ziegler, and others (outside the scope of this course).

My goal is to develop the basics of differential algebra and algebraic geometry, as well as linear algebraic groups, leading to a proof of the fundamental theorem of differential Galois theory, all the while pointing out the model theoretic point of view. Time permitting, I will also discuss other topics; for example, the connection of the differential Galois group to the monodromy group, and the Liouville-Risch-Rosenlicht theory of elementary integration. I will try to make the course accessible for those with a basic knowledge of graduate algebra (groups, rings, fields) and a modicum of model theory (on the level of 220A). If in doubt, ask me.

References.

- E. R. Kolchin, *Differential Algebra and Algebraic Groups*, Pure and Applied Mathematics, vol. 54, Academic Press, New York-London, 1973.
- A. R. Magid, *Lectures on Differential Galois Theory*, University Lecture Series, vol. 7, American Mathematical Society, Providence, RI, 1994.
- I. Kaplansky, *An Introduction to Differential Algebra*, 2nd ed., Actuelles Scientifiques et Industrielles, no. 1251, Publications de l'Institut de Mathématique de l'Université de Nancago, no. V, Hermann, Paris, 1976.
- M. van der Put, M. F. Singer, *Galois Theory of Linear Differential Equations*, Grundlehren der Mathematischen Wissenschaften, vol. 328, Springer-Verlag, Berlin, 2003.
- D. Marker, M. Messmer, A. Pillay, *Model Theory of Fields*, 2nd ed., Lecture Notes in Logic, vol. 5, Association for Symbolic Logic, La Jolla, CA; A K Peters, Ltd., Wellesley, MA, 2006.