Problem Set 5
Due Friday, May 11.

Real Analysis

Math 131A, Spring Quarter 2018

1. Do problems 11.1, 11.6 in the textbook.
2. Let (s_n) be a sequence. A real number s is called an *accumulation point* of (s_n) if for each $\varepsilon > 0$ there are infinitely many n such that $|s_n - s| < \varepsilon$. In class we proved that this is equivalent to the existence of a subsequence of (s_n) converging to s.
   (a) Prove that a bounded sequence with exactly one accumulation point must converge. Can the requirement that the sequence is bounded be dropped?
   (b) Suppose (s_n) is bounded. The Bolzano-Weierstrass Theorem states that (s_n) has at least one accumulation point. The *limit superior* of (s_n) is defined as
   $$\limsup s_n := \sup\big\{s : s \text{ is an accumulation point of } (s_n)\big\}.$$
   Show that if $\limsup s_n < a$ then there is some n_0 such that $s_n < a$ for all $n \geq n_0$.
3. Do 14.1, 14.5, 14.6, 14.8, 14.14 in the textbook.
4. Extra credit: 14.7 in the textbook.