Problem Set 1
Due Friday, April 10.

Real Analysis

Math 131A, Spring Quarter 2015

1. Do problems 1.5, 1.8, 1.11, 2.4, 3.5, 3.8 in the textbook.
2. Prove that $(1+x)^n \geq 1+nx$ if $x \in \mathbb{R}$ with $1+x>0$, for all natural numbers $n \geq 1$. (Bernoulli's Inequality.)
3. By induction on n, show that $2^n > n^2$ for all natural numbers $n \geq 5$.
4. Prove that $1^2+2^2+\cdots+n^2 = \frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n = \frac{1}{6}n(n+1)(2n+1)$, for all natural numbers $n \geq 1$.
5. Here is a proposed proof for the statement "all cats have the same color". We proceed as follows: By induction on n we show that in any set of n cats, there are no two cats with a different color. *Base step:* A set consisting of one cat only clearly satisfies the claim. *Inductive step:* Suppose the theorem has been proven for all sets of n cats. Consider a set of $n+1$ cats. Take one cat out of the set; call it "first cat". By the inductive hypothesis, the rest of the cats (consisting of n cats), are of the same color, say color x. Take another cat away; call it "second cat". Remember that the rest of the set has $n-1$ cats of color x. Now return the first cat to the set. The set has now n cats, $n-1$ of which are color x; the first cat has some unknown color. Since by inductive hypothesis a set of cats with n elements has just one color, the first cat must also have color x. Now bring the second cat back in, and we get back our set of $n+1$ cats, all of which have color x. To summarize, we assumed any n cats were of color x, and we have proven that any $n+1$ cats are of color x also. *Conclusion:* By virtue of mathematical induction, all cats in the world are of the same color. Since this conclusion is obviously false, only three possibilities are available: the principle of mathematical induction is false or inapplicable to cats, logic is false or inapplicable to cats, or there is an error in the proof. Which one? Explain why!