

Problem Set 1  
Due Friday, April 11.

*Mathematical Logic*

Math 114L, Spring Quarter 2008

1. (30 pt.) Exercise 2 on p. 19 of the textbook.
2. (30 pt.) Exercise 3 on p. 19 of the textbook.
3. (20 pt.) Read Theorem 0B and its proof on p. 6 of the textbook. Use this theorem to show that the set of all expressions is countable.
4. (20 pt.) Give a proof of the fact that every subset of a countable set is countable. Then use this to show that the set of all wffs is countable.
5. (30 pt. extra credit.) Consider the set of all finite strings consisting of the symbols  $M$ ,  $U$  and  $I$ . Let  $P$  be the set of all such strings which are built up from the string  $MI$  by finitely many applications of the following rules. Here  $x, y$  are strings, and concatenated strings are denoted by writing them one after the other.
  - (P1) If  $xI \in P$ , then  $xIU \in P$ ;
  - (P2) if  $Mx \in P$ , then  $Mxx \in P$ ;
  - (P3) if  $xIIIy \in P$ , then  $xUy \in P$ ;
  - (P4) if  $xUUy \in P$ , then  $xy \in P$ .

Prove the following claims:

- (a)  $MUUIU \in P$ .
- (b)  $MU \notin P$ .