1. Do problems 3.3.9, 3.3.11, 3.3.14, 3.3.19, 3.3.33, 3.3.38 in the textbook.

2. Prove that the Binomial Theorem holds in any commutative ring \( R \) with identity: if \( n \geq 1 \) and \( a, b \in R \), then

\[
(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k.
\]

Here we set \( a^0 := 1 \) for any \( a \in R \), and

\[
\binom{n}{k} = \frac{n!}{k!(n-k)!} = n(n-1) \cdots (n-k+1) \quad \text{for } 0 \leq k \leq n.
\]

Also, recall that for a natural number \( n \) and \( a \in R \), \( na \) denotes the element \( a + a + \cdots + a \) (\( n \) many \( a \)'s) of \( R \).

Hint: you may use that

\[
\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k} \quad \text{for all } 1 \leq k \leq n.
\]

3. Let \( R \) be a commutative ring with identity. An element \( a \) of \( R \) is called \textbf{nilpotent} if \( a^n = 0 \) for some \( n \geq 1 \).

   (a) Determine the nilpotent elements of \( \mathbb{Z} \).

   (b) Determine the nilpotent elements of \( \mathbb{Z}_{12} \).

   (c) Let \( a, b, c \in R \) where \( a \) and \( b \) are nilpotent. Show that then \( a + b \) and \( ac \) are nilpotent. (Hint: you may use Problem 2.)