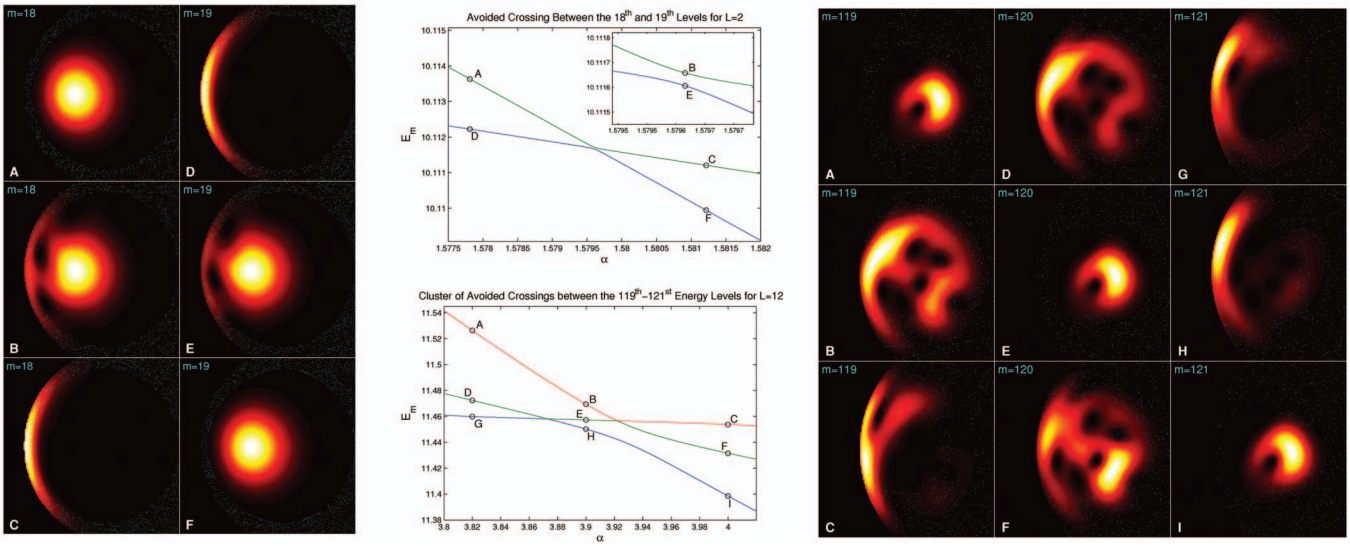

FIG. 1. (Color). (Enhanced online.)

Avoided level crossings in the quantization of a mixed regular-chaotic system

Thomas Mainiero
Department of Physics, California Institute of Technology, Pasadena, California 91125, USA
Mason A. Porter
Mathematical Institute,University of Oxford, Oxford OX1 3LB, United Kingdom
(Received 23 August 2007; published online 27 December 2007)
[DOI: 10.1063/1.2786009]

Quantum chaos entails the investigation of the quantization of classical Hamiltonian systems with chaotic dynamics.[1,2] It is particularly interesting to consider the quantization of systems with mixed regular (integrable) and chaotic dynamics, as typical classical Hamiltonian systems possess mixed dynamics, with islands of integrability situated in a chaotic sea.

One particularly appealing example consists of a particle on a ring coupled linearly to a harmonic oscillator on one section of the ring.[3] The classical system possesses large, well-divided regions of integrable and chaotic dynamics. In the present short paper, we study the quantum signatures of these dynamics.[4]

The eigenvalues of the Hamiltonian describing the quantum system can approach each other very closely, exhibiting so-called "avoided crossings," as the coupling strength α is varied. The top center figure displays what is known as a "sharp" avoided crossing. The figure's inset provides a magnification of the closest approach of the levels. Adjusting the length L of the uncoupled region to increase the fraction of the classical system's phase space that is chaotic yields broader avoided crossings in which some eigenvalues are nearly degenerate over a longer range of coupling strengths. The bottom center figure illustrates an interacting cluster of such "broad" avoided crossings.

To compare the quantum dynamics to corresponding classical surfaces of section (SOS), we show Husimi distributions[5] on the left (right) for the top (bottom) center plot. We depict these distributions as contour plots that range from zero probability density in black to higher probability densities in progressively brighter shades of red. For each Husimi distribution, we overlay (in blue) the corresponding SOS, whose empty spaces indicate regions of integrability. The left figure shows a continuous and complete exchange of structure between the participating levels in a narrow window around their closest approach; this exchange occurs between a state clearly localized in an integrable region and one localized around the edge of the chaotic sea. As α is varied through the avoided crossing, probability density continuously flows through bridges between the integrable and chaotic regions. The right figure shows a more complicated exchange of structure in the Husimi distributions near the cluster of avoided level crossings. Clusters that include broad avoided crossings often involve complex level interactions, leading to a "mixing" of initial Husimi distributions as well as delocalization of the probability density in one or more of the participating levels.

The study of avoided level crossings in the spectra of quantum Hamiltonians whose classical counterparts exhibit mixed regular-chaotic dynamics reveals important information about the quantum signatures of such dynamics. Among other topics, one can study delocalization phenomena and exchange of structure in Husimi distributions and compare the quantum dynamics one observes to the dynamics of corresponding classical systems at the same parameter values.

[1]M. C. Gutzwiller, *Chaos in Classical and Quantum Mechanics* (Springer-Verlag, New York, 1990).
[2]F. Haake, *Quantum Signatures of Chaos*, 2nd ed. (Springer-Verlag, Berlin, Germany, 2001).
[3]S. De Bièvre, P. E. Parris, and A. Silvius, Physica D **208**, 96 (2005).
[4]T. Mainiero and M. A. Porter, "Quantization of a free particle interacting linearly with a harmonic oscillator," Chaos (to be published), arXiv:nlin/0702025.
[5]A. Bäcker, S. Fürstberger, and R. Schubert, Phys. Rev. E **70**, 036204, (2004).

