Math 168, Networks, UCLA, Spring 2018
Problem Sheet 4

(submit to CCLE by 30 April 2018 at 5:00 pm)

1. *Reading.* Read Chapter 14 of Newman's book.

2. *Revisiting some material from lecture.*

    (a) Go through the argument for calculating the global clustering coefficient for the Newman–Watts variant (i.e., adding shortcuts) of the Watts–Strogatz model, and write it down carefully.

3. Do Problem 15.1 of Newman's book.

4. Do Problem 15.2 of Newman's book.

5. *Simulating Watts–Strogatz networks or its variants.*

    (a) Using numerical simulations of either Watts–Strogatz networks or Newman–Watts networks (or both) for many values of p, illustrate via a plot like the one I showed in lectures the transition between large worlds and small worlds. Your vertical axis should include both a clustering coefficient and mean geodesic path length. Try this for different numbers of nodes N. What do you observe as N becomes larger? For a given value of N, what does the plot look like if you show the results for a single realization of a network for each value of p? What does it look like if the vertical axis has sample means over some reasonably large number of realizations of a WS and/or NW ensemble for each value of p?

    Note: There is numerous code online for generating WS networks or variants thereof. Here is one of them: `http://uk.mathworks.com/help/matlab/math/build-watts-strogatz-small-world-graph-model.html`. If you use existing code, explicitly cite the one that you use.

    (b) Is the Watts–Strogatz model a good model for a social network? Why or why not?

6. *More on rewiring and adding shortcuts in WS and NW networks.* In the 4/16 lecture, one student asked astutely about how exactly one specifies the edge rewiring (for the WS model) or adding of shortcut edges (for the NW variant of the WS model), and in particular she asked what we do in situations in which the proposed "new" edge already exists. This is an excellent and important question, as it is necessary to specify this in practice to have a well-defined model. In this problem, I want you to explore this issue.

    (a) Using numerical computations of the WS model, explore at least two different choices of how do deal with this issue and compare your results from these different choices.

    (b) Repeat part (a), but now using the NW model. Do you have any different observations here than you did in part (a)?

    Note: There is a type of random graph (*configuration models*, which we will discuss later in the course) where one needs to worry about what to do about self-edges and multi-edges. If you want a preview about this topic, take a look at the following paper (including it's Seussian ending) by Bailey Fosdick et al.: `https://arxiv.org/abs/1608.00607` (*SIAM Review*, 2018, in press). Newman's textbook discusses configuration models, and you may find it helpful to peruse this paper while you read Newman's discussion of this topic.

    Note 2: Among the things that this issue illustrates is that the details are important. You will read many papers in network science (and almost every other topic) in which, when you try to implement a model from a paper, you find that some detail(s) has not been stated with full precision. There will be times — probably many of them — where different choices (even of things that may seem minor at first glance) of how to implement something in practice will lead to nontrivial differences in a model. This is also a very familiar experience for graduate students when they try to duplicate somebody else's work.

7. *More on group projects.* [moved from homework 3]

    (a) Write a short abstract and (roughly) half-page outline of what you plan to do for your project. (These plans will almost certainly change, so this should just reflect your current view.) The same thing will be submitted for each group member.

(b) For these plans, indicate briefly what each person plans to do for the project. (Surely, this will also change.) The same thing will be submitted for each group member.

(c) Each person should *individually* indicate what they hope to get out of the project and what they hope to learn with the project. (A couple of sentences will suffice.)

(d) Indicate what data you hope to use — or plan to use, if you have gotten that far — for the project. I suggest you spend a bit of time on this to see if it's feasible (with available and sufficiently clean data). This is the type of thing that can cause plans to change and is a key topic for us to discuss when I meet with each group.

Note: I am happy to arrange a meeting with groups before this assignment is due, especially with the delay in this problem.