Math 168, Networks, UCLA, Spring 2018
Problem Sheet 5

(submit to CCLE by Monday 7 May 2018 at 5:00 pm)

1. *Reading.* Read Sections 7.2–7.8, 7.11–7.13, 8.5, and 8.7 of Newman's book.

2. *Brief review of the fundamentals.* Do Problem 6.2 in Newman's book.

3. *Practice with graph Laplacians and using linear algebra to study networks.* Let $G = (V, E)$ be a connected graph with a node set V and edge set E. Let $\mathbf{A}$ denote its associated adjacency matrix.

   (a) Suppose that $\lambda_1, \ldots, \lambda_N$ are the eigenvalues of the graph's combinatorial graph Laplacian $\mathbf{L}$, where we order the eigenvalues so that $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$. Show that $\lambda_2 > 0$ (which means that there is a so-called *spectral gap*).

   (b) Let $\mathbf{L}_{\mathrm{rw}} := D^{-1}\mathbf{A}$ denote the *random-walk Laplacian* of the graph G. Show that its eigenvalues μ_i satisfy $|\mu_i| \leq 1$ for all $i \in \{1, \ldots, N\}$.

   (c) Let ν_i denote the eigenvalues of $\mathbf{A}$, and suppose that G is a k-regular graph. Show that $|\nu_i| \leq k$ for all $i \in \{1, \ldots, N\}$.

   Note: Based on the results of Quiz 1, I think it's important to emphasize these ideas and to practice more with them. Hopefully, this problem will help with that.

4. Do Problem 14.1 of Newman's book.

5. Do Problem 14.2 of Newman's book.

6. Do Problem 14.3 of Newman's book.

7. *BA model and numerical computations.* Conduct numerical simulations of the BA model. [You can easily find code online.] What behavior do you observe? What kinds of finite-size effects do you observe? Make sure to specify what you choose as an initial seed network. Additionally, comment on the suitability of the BA model as a model for the structure of the World Wide Web.