Math 168, Networks, UCLA, Spring 2018
Make-Up Midterm (31 May 2018)

Note: For this exam, you may use hard copies of Newman's book, your notes, and your homework. You may not use anything electronic.

Note 2: Include your reasoning and/or derivations (as appropriate) in your answers to ensure that they are complete.

Note 3: Please include your name and student ID number on each page of the exam.

1. (40 points) Consider the network G in Fig. 1.

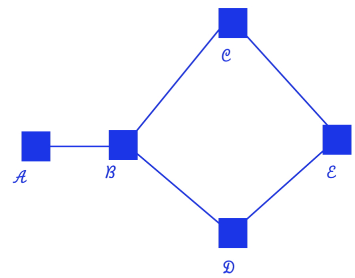

FIG. 1: A network.

a. (5 points) Write down the adjacency matrix $\mathbf{A}$ and the combinatorial Laplacian matrix $\mathbf{L}$ of G. How many 0 eigenvalues does $\mathbf{L}$ have?

b. (7 points) Determine the degree distribution of G.

c. (7 points) Calculate the geodesic node betweenness centrality of each node of G.

d. (7 points) Calculate the local clustering coefficient of each node of G.

e. (7 points) Define and precisely distinguish the Erdős–Rényi models $\mathcal{G}(N, m)$ and $\mathcal{G}(N, p)$. Could G have been generated from the model $\mathcal{G}(N, p)$? Why or why not?

f. (7 points) Suppose that you partition G into two communities, such that nodes A and B are in one community and nodes C, D, and E are in the other. Calculate the modularity Q (with the standard configuration-model-like choice of null-model P_{ij}) of this partition.

2. (40 points) Consider the following variation of the Watts–Strogatz (WS) small-world model: We again have a ring of N nodes, and each node is adjacent to its b nearest neighbors (where b is even) in the ring. We again add shortcuts to the network with probability p for each edge around the ring. However, instead of connecting random pairs of networks, we add a hub node (this is node $N+1$ in this network) in the center, and each shortcut connects the hub to a node chosen uniformly at random node from the ring.

   a. (15 points) Show (non-rigorous calculations are ok) that the mean geodesic distance between two nodes in this network, as $N \to \infty$, is

   $$\ell = \frac{2(b^2 p + 1)}{b^2 p}\,.$$

   When doing this calculation, what (if anything) does one need to say about the relative values of b and N?

   b. (10 points) For $p = p^* \in (0, 1)$ (i.e., consider a value other than 0 or 1), briefly explain the general form of the adjacency matrices associated to (1) this small-world model and (2) the Newman–Watts (NW) variant of the WS small-world model.

   c. (5 points) State (with a brief justification) at least one respect in which this new small-world model is a good model for a transportation network and at least one respect in which this model is not a good model for a transportation network.

   d. (10 points) Suppose that you are doing numerical computations of the mean geodesic distance ℓ between nodes and a global clustering coefficient C for the NW model, with different values of the parameter p. Supposing that you look at exactly one NW network for each value of $p \in [0, 1]$, indicate in a sketch what C and ℓ might look like as a function of p. How would this plot change if you average your results over 100 different NW networks for each value of p? How would the latter plot change if you instead consider the WS model?

3. (20 points) Consider the random-graph model $\mathcal{G}(N, p)$ with mean degree c.

   a. (8 points) Show in the limit of large N that the expected number of 3-cliques in the network is $c^3/6$. What, if anything, does this imply about $\mathcal{G}(N, p)$ networks?

   b. (7 points) Show that the expected number of connected triples is $\frac{1}{2}Nc^2$.

   c. (5 points) What do the results of parts (a) and (b) indicate about the global clustering coefficient of $\mathcal{G}(N, p)$ networks?