285J, L. Vese

**Assignment 3:** Due on Friday, June 10.

- Let $g \in C^1(R)$ be a function, with $g' > 0$. Let $v = g(u)$. If $u$ satisfies
  \[
  \frac{\partial u}{\partial t} = |\nabla u| G(\text{curv}(u)),
  \]
  so does $v$ (contrast invariance or geometric).

- Let $u : R^2 \to R$. The upper level set of $u$ at level $\lambda \in R$ is the set $\chi_\lambda(u) = \{x \in R^2 : u(x) \geq \lambda\}$. Show that $u$ can be retrieved by the reconstruction formula
  \[
  u(x) = \sup\{\lambda : x \in \chi_\lambda(u)\}.
  \]

- Let $u, v : R^2 \to R$. Assume that $u$ and $v$ have the same level sets, that is for all $\lambda \in R$, there is $\mu \in R$ such that $\chi_\lambda(u) = \chi_\mu(v)$. Let us define $g$ by
  \[
  g(\lambda) = \sup \{\mu : \chi_\lambda(u) = \chi_\mu(v)\}. \tag{1}
  \]
  Then $g$ is nondecreasing and $v = g \circ u$.

- Let $u$ be sufficiently smooth and satisfy
  \[
  \frac{\partial u}{\partial t} = |\nabla u| G(\text{curv}u),
  \]
  where $\text{curv}u = \text{div} \left( \frac{\nabla u}{|\nabla u|} \right)$ is the curvature operator, and $G$ is a function such that $kG(k) \geq 0$. Show that this flow decreases the total variation of $u$ in time.