Consider in two dimensions \( f \in L^2(\Omega) \), and \( u(\cdot, \lambda) \) the unique minimizer of

\[
F(u) = \lambda \int_{\Omega} |Du|dx + \frac{1}{2} \int_{\Omega} |u - f|^2dx,
\]

with \( \lambda \geq 0 \).

We know that \( u(\cdot, \lambda) \) necessarily satisfies the Euler-Lagrange equation (formally)

\[
\text{(E-L)} \quad \begin{cases} 
    u(x, \lambda) - f(x) = \lambda \text{div} \left( \frac{Du(x, \lambda)}{|Du(x, \lambda)|} \right) \text{ in } \Omega, \\
    \frac{Du(x, \lambda)}{|Du(x, \lambda)|} \cdot \vec{n}(x) = 0 \text{ on } \partial \Omega,
\end{cases}
\]

where \( \vec{n}(x) \) denotes the exterior unit normal to \( \partial \Omega \) at the point \( x \in \partial \Omega \).

In the above relations, assume for simplicity that all expressions are well defined, that \( u(\cdot, \lambda) \) is a Sobolev function (for instance in \( W^{1,1}(\Omega) \cap L^2(\Omega) \)), and that \( Du(x, \lambda) = \nabla u(x, \lambda) \) denotes the usual spatial gradient (in the distributional sense, and not as a measure).

**Problems:**

1. Show that the \( L^2 \)-norm of \( u(\cdot, \lambda) \), given by \( \sqrt{\int_{\Omega} (u(x, \lambda))^2dx} \) is bounded by a constant independent of \( \lambda \).

2. Show, using the above E-L equation and above B.C., that

\[
\int_{\Omega} u(x, \lambda)dx = \int_{\Omega} f(x)dx.
\]

3. Show that \( u(\cdot, \lambda) \) converges in the \( L^1(\Omega) \) – strong topology to the average of the initial data. In other words, show that

\[
\lim_{\lambda \to \infty} \int_{\Omega} \left| u(x, \lambda) - \frac{\int_{\Omega} f(x)dx}{|\Omega|} \right| dx = 0.
\]