FALL 2001 - MATH 285J SEMINAR: Applied Mathematics
Variational Methods & PDE's for Image Analysis and Curve Evolution

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PLAN & BIBLIOGRAPHY (tentative)

0 Introduction

1. Image processing tasks
2. Applications
3. Representation of images in the continuous and discrete case, gray level and color images, degradation operators (blur), noise, gradient magnitude

1 Image Restoration

1. Variational methods: Total Variation of Rudin-Osher-Fatemi and related models [1], [2], [3], [4]
2. Anisotropic diffusion of Perona-Malik [5], nonlinear diffusion [6], [7], image-enhancement using shock filters of Osher-Rudin [8]
3. "Fundamental equation of image analysis" of Alvarez-Guichard-Lions-Morel and axiomatization [9]
4. Other related topics (if time permits): half-quadratic regularization [4], [10], [2], blind deconvolution Chan-Wong [11], higher order variational models for image restoration [2], [12].

The following topics will be introduced and discussed: functions of bounded variation and functionals on the BV space, existence and uniqueness, Euler-Lagrange equation, discretization by finite differences.

2 Image Segmentation

1. Mumford and Shah model for piecewise-smooth optimal approximations [13]
2. The weak formulation for the Mumford and Shah problem by Dal Maso-Morel-Solimini [14]
3. Approximations to the weak formulation by finite differences Chambolle [16], [17], elliptic approximations by $\Gamma-$ convergence by Ambrosio-Tortorelli [15]
4. Region growing and merging [18], other numerical considerations

3 Curve and Surface Evolution in Image Processing

1. Geometric curve and surface evolution (introduction)
2. Level set formulation of Osher-Sethian [19]
3. Viscosity solutions [20], [21]
4. Object detection by snakes [22], active contours and level sets [23], [24]

Books and Lecture Notes

• J.-M. Morel and S. Solimini, *Variational Methods in Image Segmentation*, PNLDE 14, Birkhäuser Boston 1995.

• J. Weickert, *Anisotropic Diffusion and Image Processing*, Teubner-Verlag, Stuttgart, 1998.

• G. Sapiro, *Geometric Partial Differential Equations and Image Analysis*, Cambridge University Press, 2001.

• J. Sethian, *Level Set Methods and Fast Marching Methods Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision, and Materials Science*, Cambridge Monograph on Applied and Computational Mathematics, Cambridge University Press, 1999.

• F. Guichard and J.M. Morel, *Image Analysis and PDEs*, to appear.

• G. Aubert and P. Kornprobst, *Mathematical Problems in Image Processing*, to appear.

• S. Osher and R. Fedkiw, *Level Sets and Dynamic Implicit Surfaces*, to appear.

• A. Braides, *Approximation of Free-Discontinuity Problems*, ISAS-International School for Advanced Studies SISSA, Trieste, 1998.

References

[1] L.I. Rudin, S. Osher, E. Fatemi, *Nonlinear Total Variation Based Noise Removal Algorithms*, Physica D, 60 (1-4): 259-268 Nov 1 1992.

[2] A. Chambolle and P.L. Lions, *Image recovery via total variation minimization and related problems*, Numerische Mathematik, 76 (2): 167-188 Apr 1997.

[3] L. Vese, *A study in the BV space of a denoising-deblurring variational problem*, Applied Mathematics and Optimization, 44 (2): 131-161 Sep-Oct 2001.

[4] G. Aubert and L. Vese, *A variational method in image recovery*, SIAM Journal on Numerical Analysis, 34 (5): 1948-1979 Oct 1997.

[5] P. Perona and J. Malik, *Scale-space and edge-detection using anisotropic diffusion*, IEEE Transactions on Pattern Analysis and Machine Intelligence, 12 (7): 629-639 Jul 1990.

[6] F. Catté, P.L. Lions, J.-M. Morel and T. Coll, *Image selective smoothing and edge-detection by nonlinear diffusion*, SIAM Journal on Numerical Analysis, 29 (1): 182-193 Feb 1992.

[7] L. Alvarez, P.L. Lions, J.-M. Morel and T. Coll, *Image selective smoothing and edge-detection by nonlinear diffusion 2*, SIAM Journal on Numerical Analysis, 29 (3): 845-866 June 1992.

[8] S. Osher and L.I. Rudin, *Feature-oriented image-enhancement using shock filters*, SIAM Journal on Numerical Analysis, 27 (4): 919-940 Aug. 1990.

[9] L. Alvarez, F. Guichard, P.L. Lions and J.-M. Morel, *Axioms and fundamental equations of image processing*, Archive of Rational Mechanics and Analysis, 123 (3): 199-257 1993.

[10] Charbonnier, P.; Blanc-Feraud, L.; Aubert, G.; Barlaud, M., *Deterministic edge-preserving regularization in computed imaging*, IEEE Transactions on Image Processing, Volume: 6 Issue: 2 , Feb. 1997, Pages: 298 -311.

[11] T.F. Chan and C.K. Wong, *Total variation blind deconvolution*, IEEE Transactions on Image Processing, Volume: 7 Issue: 3 , March 1998, Pages: 370 - 375.

[12] Chan T, Marquina A, Mulet P, *High-order total variation-based image restoration*, SIAM Journal on Scientific Computing, 22 (2): 503-516 Aug 31 2000.

[13] D. Mumford and J. Shah, *Optimal approximations by piecewise smooth functions and associated variational problems*, Communications on Pure and Applied Mathematics, 42 (5): 577-685 Jul 1989.

[14] Dal Maso G., Morel J.-M., and Solimini S., *A variational method in image segmentation - existence and approximation results*, Acta Mathematica, 168 (1-2): 89-151 1992.

[15] L. Ambrosio and V.M. Tortorelli, *On the approximations of free discontinuity problems*, Bollettino de la Unione Matematica Italiana, 6B (1): 105-123 Mar 1992.

[16] A. Chambolle, *Finite-differences discretizations of the Mumford-Shah functional*, RAIRO-Mathematical Modelling and Numerical Analysis - Modélisation Mathématique et Analyse Numérique, 33 (2): 261-288 Mar-Apr 1999.

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