Math 270C: Assignment #6

Due on Monday, June 2nd.

[**1**] Consider the Krylov subspace $K(A, \vec{b}, p) = span\{\vec{b}, A\vec{b}, ..., A^{p-2}\vec{b}, A^{p-1}\vec{b}\}$, where A is an n by n matrix. If A is non-singular and $A^p\vec{b} \in K(A, \vec{b}, p)$ then show

(a) $A(K(A, \vec{b}, p)) \subseteq K(A, \vec{b}, p)$.

(b) there exists a solution $\vec{x} \in K(A, \vec{b}, p)$ such that $A\vec{x} = \vec{b}$.

[**2**] Consider the second equivalent version of the conjugate gradient method, where A is symmetric and positive definite, as given in the handout Algorithm [Conjugate Gradients], taken from Golub-Van Loan.

Assume now that A is only non-singular. Apply this algorithm first to solve the system $AA^T y = b$, using $\hat{p}_k$ for the directions in y_k. Then by change of variables $x_k = A^T y_k$ and $p_k = A^T \hat{p}_k$, recover the CGNE algorithm (see also Golub-Van Loan).

[**3**] Implement the ADI method for the model problem from [4], Assignment #3. Use again $h = 1/32$, and $\omega_{opt} = 2\frac{\sin(\pi h)}{h^2} \approx 200.7391$. Plot the error versus iterations. Comment your results.

(on the web page you will find a pseudo-code for "Symmetric, Tridiagonal, Positive Definite System Solver")