Math 270C: Assignment #4

Due on Monday, May 12.

[**1**] Show the Lemma from the course:

$$'' \lim_{m\to\infty} \|A^m\| = 0 \text{ if and only if } \rho(A) < 1 \; ''$$

for real matrices A with a complete set of linearly independent eigenvectors (use a different proof).

[**2**] Let A be an $n \times n$ symmetric positive definite matrix. Consider the $A - norm$ of a vector $\vec{x} \in R^n$ defined by

$$\|\vec{x}\|_A^2 = \langle \vec{x}, A\vec{x} \rangle.$$

(a) If A is not positive definite, what conditions that define a vector norm are not satisfied by $\|\vec{x}\|_A$?

(b) Is symmetry required to insure that $\|\vec{x}\|_A$ is a vector norm ?

[**3**] Assume A is an $n \times n$ symmetric positive definite matrix. Let $\{\vec{p}_j\}$, $j = 1...M$ be a set of $A-$orthogonal vectors, e.g. $\langle \vec{p}_j, A\vec{p}_i \rangle = 0$ if $i \neq j$. Show that if $\vec{x}$ is a solution to $A\vec{x} = \vec{b}$, then the coefficients $\{c_j\}$, $j = 1...M$ that minimize

$$\|\vec{x} - \sum c_j \vec{p}_j\|_A$$

are given by

$$c_j = \frac{\langle \vec{b}, \vec{p}_j \rangle}{\langle \vec{p}_j, A\vec{p}_j \rangle}.$$

(Tip: $\vec{x} = A^{-1}\vec{b}$)

[**4**] Implement the gradient descent method for the model problem from [4], assignment 3. Use again 300 iterations and $h = 1/32$. Plot the error versus iterations. Comment your results.