

Math 270C: Assignment 4

Due Monday, February 13, 2006

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[1] Show the Lemma from the course:

$$\lim_{m \rightarrow \infty} \|A^m\| = 0 \text{ if and only if } \rho(A) < 1$$

for real matrices A with a complete set of linearly independent eigenvectors (use a different proof).

[2] Let A be an $n \times n$ symmetric positive definite matrix. Consider the A -norm of a vector $\vec{x} \in R^n$ defined by

$$\|\vec{x}\|_A^2 = \langle \vec{x}, A\vec{x} \rangle.$$

(a) If A is not positive definite, what conditions that define a vector norm are not satisfied by $\|\vec{x}\|_A$?

(b) Is symmetry required to insure that $\|\vec{x}\|_A$ is a vector norm?

[3] Assume A is an $n \times n$ symmetric positive definite matrix. Let $\{\vec{p}_j\}$, $j = 1 \dots M$ be a set of A -orthogonal vectors, e.g. $\langle \vec{p}_j, A\vec{p}_i \rangle = 0$ if $i \neq j$. Show that if \vec{x} is a solution to $A\vec{x} = \vec{b}$, then the coefficients $\{c_j\}$, $j = 1 \dots M$ that minimize

$$\|\vec{x} - \sum c_j \vec{p}_j\|_A$$

are given by

$$c_j = \frac{\langle \vec{b}, \vec{p}_j \rangle}{\langle \vec{p}_j, A\vec{p}_j \rangle}.$$

(Tip: $\vec{x} = A^{-1}\vec{b}$)

[4] Implement the gradient descent method for the model problem from [4], assignment 3. Use again 300 iterations and $h = 1/32$. Plot the error versus iterations. Comment your results.