

Math 270C: Assignment 7

(last assignment)

Due Monday March 14 or Wednesday, March 16, 2005

(no late homework accepted)

Instructor: Luminita Vese

[1] Give the analytic solution to

$$-(3u_{xx} + 2u_{yy}) = \sin((3/2)\pi x)\sin(4\pi y) + \sin((5/2)\pi x)\sin(2\pi y),$$

for $(x, y) \in [0, 2] \times [0, 1]$ and $u(x, y) = 0$ on the boundary of this domain.

[2] Suppose a uniform mesh that has M panels in the x direction and N panels in the y direction for the domain $[0, 2] \times [0, 1]$ is used. If u_{xx} and u_{yy} are approximated using standard 2nd order, 3-point difference formulas, what is the solution of the discrete equations that approximates the problem in [1] ?

[3] Use preconditioned conjugate gradients to compute the solution to the linear set of equations arising from a discretization of

$$\operatorname{div}(a(x, y)\nabla u) = f \text{ for } (x, y) \in \Omega,$$

$$u = 0 \text{ on } \partial\Omega,$$

where $\Omega = [-1, 1] \times [-1, 1]$, $f(x, y) = \sin(\pi x)\sin(\pi y)$, and $a(x, y)$ is a discontinuous function given by

$$a(x, y) = \begin{cases} \alpha_0 & \text{if } \sqrt{x^2 + y^2} \leq \frac{1}{2}, \\ \alpha_1 & \text{if } \sqrt{x^2 + y^2} > \frac{1}{2}. \end{cases}$$

[a] Use values of $\alpha_0 = 1$ and $\alpha_1 = 10, 100, 1000$. Use 100 panels in each direction and stop your iteration when the size of the residual is less than h^2 . Record the number of iterations for each set of coefficients.

[b] Use values of $\alpha_0 = 10, 100, 1000$ and $\alpha_1 = 1$. Use 100 panels in each direction and stop your iteration when the size of the residual is less than h^2 . Record the number of iterations for each set of coefficients.

[c] How does the number of iterations depend upon the ratio $\frac{\alpha_0}{\alpha_1}$?

(give the discretization scheme of your choice, properties of the system matrix, etc. Each time plot the size of the residual function of iterations, and the final solution as a surface.)