Math 270C: Assignment 6

Due Friday, March 4, 2005

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[**1**] Consider the Krylov subspace $K(A, \vec{b}, p) = span\{\vec{b}, A\vec{b}, ..., A^{p-2}\vec{b}, A^{p-1}\vec{b}\}$, where A is an n by n matrix. If A is non-singular and $A^p\vec{b} \in K(A, \vec{b}, p)$ then show

(a) $A(K(A, \vec{b}, p)) \subseteq K(A, \vec{b}, p)$.

(b) there exists a solution $\vec{x} \in K(A, \vec{b}, p)$ such that $A\vec{x} = \vec{b}$.

[**2**]

(a) Verify the Sherman-Morrison-Woodbury formula

$$(A + UV^t)^{-1} = A^{-1} - A^{-1}U(I + V^tA^{-1}U)^{-1}V^tA^{-1}$$

(we assume that both A and $(I + V^tA^{-1}U)$ are non-singular).

(b) Let A be an n by n non-singular matrix and A' be a matrix that differs from A in a single element. What are the matrices U and V^t so that $A' = A + UV^t$? (and hence one might use the S-M-W formula to compute $(A')^{-1}$?)

[**3**] Implement the ADI method for the model problem from [4], assignment 3. Use again $h = 1/32$, and $\omega_{opt} = 2\frac{\sin(\pi h)}{h^2} \approx 200.7391$. Plot the error versus iterations. Comment your results.

(on the web page you will find a pseudo-code for "Symmetric, Tridiagonal, Positive Definite System Solver")