Math 270C: Assignment 5
Due Friday, February 18, 2005
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[1] Assume that the $n \times n$, symmetric, positive definite matrix $A$ has the eigenvalues $\lambda_i, i = 1, \ldots, n$, with corresponding l.i. eigenvectors $v_i, i = 1, \ldots, n$.
(a) Find the eigenvalues of $P_k(A)$, where $P_k(A) = \gamma_0 I + \gamma_1 A + \ldots + \gamma_k A^k$, with $\gamma_i$ constants.
(b) Show that any eigenvector $v_i$ of $A$ is also an eigenvector of $P_k(A)$.
(c) Assume that $x \in \mathbb{R}^n$ is given by $x = \sum_{i=1}^n \xi_i v_i$, for some constants $\xi_i$. Assume now that the eigenvectors $v_i$ are orthonormal. Show that $\|x\|^2_A = \sum_{i=1}^n \lambda_i \xi_i^2$.

[2] Let $\{\lambda_i, v_i\}, i = 1, \ldots, n$ be the eigenpairs of $A$. Show that the eigenvalues and eigenvectors of

$$[I + P_k(A)A]^T A[I + P_k(A)A]$$

are $\lambda_i[1 + \lambda_i P_k(\lambda_i)]^2$ and $v_i$, respectively, where $P_k$ is defined as before.