Math 270C: Assignment 5

Due Friday, February 18, 2005

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[**1**] Assume that the $n \times n$, symmetric, positive definite matrix A has the eigenvalues λ_i, $i = 1, ..., n$, with corresponding l.i. eigenvectors v_i, $i = 1, ..., n$.

(a) Find the eigenvalues of $P_k(A)$, where $P_k(A) = \gamma_0 I + \gamma_1 A + ... + \gamma_k A^k$, with γ_i constants.

(b) Show that any eigenvector v_i of A is also an eigenvector of $P_k(A)$.

(c) Assume that $x \in R^n$ is given by $x = \sum_{i=1}^n \xi_i v_i$, for some constants ξ_i. Assume now that the eigenvectos v_i are orthonormal. Show that $\|x\|_A^2 = \sum_{i=1}^n \lambda_i \xi_i^2$.

[**2**] Let $\{\lambda_i, v_i\}$, $i = 1, ..., n$ be the eigenpairs of A. Show that the eigenvalues and eigenvectors of

$$[I + P_k(A)A]^T A [I + P_k(A)A]$$

are $\lambda_i [1 + \lambda_i P_k(\lambda_i)]^2$ and v_i, respectively, where P_k is defined as before.

[**3**] Implement the conjugate gradient method for the model problem from [4], assignment 3. Use again $h = 1/32$. Plot the error versus iterations. Comment your results.