269C. Spring. Instructor: L. Vese

Computational Project #2

(I) Consider the polynomial function on the unit square defined by

$$u(x, y) = 2^{4p} x^p (1 - x)^p y^p (1 - y)^p, \quad (x, y) \in [0, 1] \times [0, 1],$$

where p is a positive integer.

(a) Check that $u = 0$ on $\partial\Omega$.

(b) Compute $-\triangle u(x, y) + u(x, y)$ and denote the result by $f(x, y)$.

(c) Consider the equation on the unit square $\Omega = (0, 1) \times (0, 1)$:

$$-\triangle u + u = f \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega,$$

and use P_1 elements to approximate its solution. You can compare the numerical solution with the exact analytic solution (you can choose $p \leq 10$ or around 10; the accuracy of the approximation will depend on the choice of p).

(II) Use P_1 elements to approximate the solution of

$$-\triangle u + u = \sin(2\pi(x + y)), \ (x, y) \in \Omega = \text{unit square}$$

with the following boundary conditions:

Case (a) $u = 0$ for $(x, y) \in \partial\Omega$

Case (b) $u = 0$ for $(x, y) \in \partial\Omega$, $x = 0, 1$
$u_y = 0$, for $(x, y) \in \partial\Omega$, $y = 0, 1$.

Remarks

You can base the triangulations on a 10x10 grid.

• What you should turn in: the weak formulations, the linear systems, details about the discretizations, plots of the results, your computer program.

• Section 12.2 of the textbook by Johnson discusses numerical integration or quadrature formulas, helpful to discretize the load vector, if needed.