Math 269C. Vese. HW #4

[**1**] Let K be a tetrahedron with vertices a^i, $i = 1, ..., 4$, and let a^{ij} denote the midpoint on the straight line $a^i a^j$, $i < j$. Show that a function $v \in P_2(K)$ is uniquely determined by the degrees of freedom: $v(a^i)$, $v(a^{ij})$, $i, j = 1, ..., 4$, $i < j$. Show that the corresponding finite element space V_h satisfies $V_h \subset C^0(\Omega)$, assuming continuity at all degrees of freedom.

[**2**] Let K be a triangle with vertices a^i, $i = 1, 2, 3$. Suppose that $v \in P_r(K)$ and that v vanishes on the side a^2a^3. Prove that v has the form

$$v(x) = \lambda_1(x) w_{r-1}(x), \quad x \in K,$$

where $w_{r-1} \in P_{r-1}(K)$, and $\lambda_1(x)$ is the affine local basis function corresponding to the node a^1.

(for simplicity, you can assume that K is the reference triangle with vertices (0,0), (0,1) and (1,0), and that the side a^2a^3 is on one of the axes).

[**3**] Let K be a triangle with vertices a^i, $i = 1, 2, 3$, and let a^{ij}, $i < j$, denote the midpoints of the sides of K. Let a^{123} denote the center of gravity of K. Prove that $v \in P_4(K)$ is uniquely determined by the following degrees of freedom

$$\begin{aligned}
&v(a^i), \\
&\frac{\partial v}{\partial x_j}(a^i), \ i = 1, 2, 3, \ j = 1, 2, \\
&v(a^{ij}), \ i, j = 1, 2, 3, \ i < j, \\
&v(a^{123}), \ \frac{\partial v}{\partial x_j}(a^{123}), j = 1, 2,
\end{aligned}$$

(typo in Exercise 3.8 in the textbook).

Also show that the functions in the corresponding finite element space V_h are continuous, assuming continuity for all degrees of freedom.