Math 269C, Vese

First Computational Project
(due on Monday, May 13)

(i) Show that the problem

$$\begin{cases} -u''(x) + u(x) = f(x), \quad 0 < x < 1, \\ u(0) = 0, \ u'(1) = 0 \end{cases}$$

can be given a weak variational formulation.

(ii) Formulate a FEM for this problem using piecewise-linear functions. Determine the corresponding linear system of equations in the case of a uniform partition and study in particular how the boundary condition $u'(1) = 0$ is approximated by the method.

(iii) Write a computer program for the piecewise-linear FEM applied to this problem. Present calculations with $h = 0.1$ and $h = 0.2$, for $f(x) = 2+2x-x^2$. Approximate $\int f(x)\varphi_j(x)dx$ by a quadrature formula of your choice. The exact solution of the problem is $u(x) = x(2 - x)$.

Please see also Section 1.8 *Remarks on programming* and Section 12.2 *Numerical Integration (quadrature).*