

**HW #1. Math 269C (due on Friday, April 14)**

[1] Show that if  $w$  is continuous on  $[0, 1]$ , and

$$\int_0^1 wv dx = 0, \text{ for all } v \in V,$$

with  $V = \{v : [0, 1] \rightarrow \mathbb{R}, \text{ continuous, } v(0) = v(1) = 0, v' \text{ piecewise - continuous and bounded}\}$ , then  $w(x) = 0$  for  $x \in [0, 1]$ .

[2] Construct a finite-dimensional subspace  $V_h$  of  $V$  (from problem [1]) consisting of functions which are quadratic on each subinterval  $I_j$  of a partition of  $I = (0, 1)$ . How can one choose the parameters to describe such functions? Find the corresponding basis functions. Then formulate a finite element method for  $(D)$  using the space  $V_h$  and write down the corresponding linear system of equations in the case of a uniform partition. Recall that  $(D)$  is

$$-u'' = f \text{ in } (0, 1), \quad u(0) = u(1) = 0.$$

[3] Consider the BVP

$$\frac{d^4 u}{dx^4} = f, \quad 0 < x < 1, \quad u(0) = u'(0) = u(1) = u'(1) = 0.$$

(a) Show that this problem can be given the following variational formulation: Find  $u \in W$  such that

$$(u'', v'') = (f, v), \quad \forall v \in W,$$

where  $W = \{v : v \text{ and } v' \text{ are continuous on } [0, 1], v'' \text{ is piecewise-continuous and } v(0) = v'(0) = v(1) = v'(1) = 0\}$ .

(b) For  $I = [a, b]$  an interval, define

$$P_3(I) = \{v : v \text{ is a polynomial of degree } \leq 3 \text{ on } I\},$$

i.e.  $v$  has the form  $v(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$  for  $x \in I$ , and  $a_i \in \mathbb{R}$ .

Show that  $v \in P_3(I)$  is uniquely determined by the values  $v(a)$ ,  $v'(a)$ ,  $v(b)$ ,  $v'(b)$ . Find the corresponding (local) basis functions (the basis function corresponding to the value  $v(a)$  is the cubic polynomial  $v$  such that  $v(a) = 1$ ,  $v'(a) = 0$ ,  $v(b) = 0$ ,  $v'(b) = 0$ , etc.)

(c) Starting from (b) construct a finite-dimensional subspace  $W_h$  of  $W$  consisting of piecewise-cubic functions. Specify suitable parameters to describe the functions in  $W_h$  and determine the corresponding (global) basis functions.

(d) Formulate a FEM for the problem based on the space  $W_h$ . Find the corresponding linear system of equations in the case of a uniform partition.

• Note: for the question “Find the corresponding basis functions.” in problems [2], [3], you can use some software that can evaluate the integrals (since the evaluation by hand would be tedious). Or it is fine to leave out this part of the question, or just give the formulas without completely evaluating the integrals.