First computational project: due on Monday, May 11

(i) Show that the problem
\[
\begin{align*}
-\frac{d^2 u}{dx^2}(x) + u(x) &= f(x), \quad 0 < x < 1, \\
u(0) &= 0, \quad u'(1) = 0
\end{align*}
\]
can be given a weak variational formulation.

(ii) Formulate a FEM for this problem using piecewise-linear functions. Determine the corresponding linear system of equations in the case of a uniform partition and study in particular how the boundary condition \( u'(1) = 0 \) is approximated by the method.

(iii) Write a computer program for the piecewise-linear FEM applied to this problem. Present calculations with \( h = 0.1 \) and \( h = 0.2 \), for \( f(x) = 2 + 2x - x^2 \). Approximate \( \int f(x)\varphi_j(x)dx \) by a quadrature formula of your choice. The exact solution of the problem is \( u(x) = x(2 - x) \).

(see section 1.8: Remarks on programming)