Error estimates in Sobolev norms

If $\pi_h u \in P_r(\Omega)$ is the interpolant of u for triangulations T_h of Ω, satisfying the angle condition, then:

$$\|u - \pi_h u\|_{L^2(\Omega)} \leq Ch^{r+1}|u|_{H^{r+1}(\Omega)},$$

$$|u - \pi_h u|_{H^1(\Omega)} \leq Ch^r|u|_{H^{r+1}(\Omega)}.$$

If $V_h \subset H^2(\Omega)$, then we also have

$$|u - \pi_h u|_{H^2(\Omega)} \leq Ch^{r-1}|u|_{H^{r+1}(\Omega)}.$$

(C depends only on β and r; it does not depend on h or u).

If $u \in H^s(\Omega)$ with $1 \leq s \leq r+1$, then

$$\|u - \pi_h u\|_{L^2(\Omega)} \leq Ch^s|u|_{H^s(\Omega)},$$

$$\|u - \pi_h u\|_{H^1(\Omega)} \leq Ch^{s-1}|u|_{H^s(\Omega)}.$$