First computational project: due no later than May 10, 2010

(i) Show that the problem

\[ \begin{cases} -u''(x) + u(x) = f(x), & 0 < x < 1, \\ u(0) = 0, & u'(1) = 0 \end{cases} \]

can be given a weak variational formulation.

(ii) Formulate a FEM for this problem using piecewise-linear functions. Determine the corresponding linear system of equations in the case of a uniform partition and study in particular how the boundary condition \( u'(1) = 0 \) is approximated by the method.

(iii) Write a computer program for the piecewise-linear FEM applied to this problem. Present calculations with \( h = 0.1 \) and \( h = 0.2 \), for \( f(x) = 2 + 2x - x^2 \). Approximate \( \int f(x) \varphi_j(x) \, dx \) by a quadrature formula of your choice. The exact solution of the problem is \( u(x) = x(2 - x) \).

(see section 1.8: Remarks on programming)