269C: HW #5, due Friday, June 9

NOTE: Additional practice problems will be posted on the class webpage.

1. Consider the PDE (in distributional sense)

$$-\triangle u + k^2 u = f \quad \text{in } R^n.$$

Let $s \in R$. Show that, for all $f \in H^s(R^n)$, there exists a unique $u \in H^{s+2}(R^n)$, solution of the PDE, with $k \in R$, $k \neq 0$.

Hint: use the Fourier transform.

2. Let $I = [0, h]$ and let $\pi v \in P_1(I)$ be the linear interpolant that agrees with $v \in C^2(I)$ at the end points of I. Using the technique of the proof of Thm. 4.1, prove estimates for $\|v - \pi v\|_{L^\infty(I)}$ and $\|v' - (\pi v)'\|_{L^\infty(I)}$, cf. (1.12) and (1.13).

3. Using the general results from from Chapter 4, estimate the error $\|u - u_h\|_{H^2}$ for Problem 1.5 and Example 2.4.

4. Using polar coordinates (r, θ), let $\Omega = \{(r, \theta) : \ 0 < r < 1, \ 0 < \theta < \omega\}$ be a pie-shaped domain of angle ω. Prove that the function $u(r, \theta) = r^\gamma \sin(\gamma\theta)$, $\gamma = \frac{\pi}{\omega}$, satisfies: $\triangle u = 0$ in Ω, $u = 0$ on the straight parts of the boundary of Ω.