HW #4, 269C, Vese
Due Friday, May 26

NOTE: There will be no class on Monday, May 15 and Wednesday May 17. The next lecture is on Friday, May 19. We will reschedule these lectures at a later time.

1. Let K be a tetrahedron with vertices a^i, $i = 1, ..., 4$, and let a^{ij} denote the midpoint on the straight line $a^i a^j$, $i < j$. Show that a function $v \in P_2(K)$ is uniquely determined by the degrees of freedom: $v(a^i)$, $v(a^{ij})$, $i, j = 1, ..., 4$, $i < j$. Show that the corresponding finite element space V_h satisfies $V_h \subset C^0(\Omega)$.

2. Let K be a triangle in two dimensions, with vertices a^i, $i = 1, 2, 3$. Suppose that $v \in P_r(K)$ and that v vanishes on the side $a^2 a^3$. Prove that v has the form

$$v(x) = \lambda_1(x) w_{r-1}(x), \quad x \in K,$$

where $w_{r-1} \in P_{r-1}(K)$. For simplicity, you may want to assume $r \leq 3$ and use the local basis functions $\lambda_i(x)$ of $P_1(K)$.

(the proof given in class was wrong).

3. Let K be a triangle with vertices a^i, $i = 1, 2, 3$, and let a^{ij}, $i < j$, denote the midpoints of the sides of K. Let a^{123} denote the center of gravity of K. Prove that $v \in P_4(K)$ is uniquely determined by the following degrees of freedom

$$\begin{aligned}
&v(a^i), \\
&\frac{\partial v}{\partial x_j}(a^i), \ i = 1, 2, 3, \ j = 1, 2, \\
&v(a^{ij}), \ i, j = 1, 2, 3, \ i < j, \\
&v(a^{123}), \ \frac{\partial v}{\partial x_j}(a^{123}), j = 1, 2,
\end{aligned}$$

Also show that the functions in the corresponding finite element V_h are continuous.